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Dipole Moments of Black Holes and String States

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ABSTRACT

As a further test of the conjectured equivalence of string states and extremal black holes, we compute the dipole moments of black holes with arbitrary spin and superspin in $D = 4, N = 4$ supergravity coupled to 22 vector multiplets and compare them with the dipole moments of states in the heterotic string on T^6 or the Type IIA string on $K3 \times T^2$. Starting from a purely bosonic black hole with Kerr angular momentum L , the superpartners are generated by acting with fermion zero modes, thus filling out the complete supermultiplet. L is then identified with the superspin. On the heterotic side, elementary states belong only to short or long multiplets, but Type IIA elementary states can belong to intermediate multiplets as well. We find that the black hole gyromagnetic ratios are in perfect agreement with the string states not only for the BPS states belonging to short multiplets but also for those belonging to intermediate multiplets. In fact, these intermediate multiplets provide a stronger test of the black-hole/string-state equivalence because the gyromagnetic ratios are not determined by supersymmetry alone, in contrast to those of the short multiplets. We even find agreement between the non-supersymmetric (but still extremal) black holes and non-BPS string states belonging to long supermultiplets. In addition to magnetic dipole moments we also find electric dipole moments even for purely electrically charged black holes. The electric dipole moments of the corresponding string states have not yet been calculated directly but are consistent with heterotic/Type IIA duality.

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1 Introduction

Inspired by earlier work on black holes in string theory [1], it was conjectured in [2] that massive BPS string states are described by extreme black holes. By focusing on the elementary electrically charged states in the context of the toroidally compactified heterotic string, results from perturbative string theory were then used to test the conjecture. The identification was shown to be consistent with the mass and charge assignments in [2] and with the spin and supermultiplet structures in [3]. A remaining check concerns the gyromagnetic ratios, which is the subject of the present paper⁴. We shall again work with the four-dimensional $N = 4$ theory which may be regarded either as a heterotic string compactified on T^6 or else, by virtue of string/string duality [4, 5, 6, 7], as a Type IIA string compactified on $K3 \times T^2$ [8, 9]. In such an $N = 4$ theory, elementary states belong to short, intermediate or long supermultiplets with dimensions $16(2L + 1)$, $64(2L + 1)$ or $256(2L + 1)$, respectively, where L is the superspin. On the heterotic side, elementary states belong only to short or long multiplets, but Type IIA elementary states can also belong to intermediate multiplets. We shall determine both the electric and magnetic dipole moments of black hole solutions with arbitrary superspin and find perfect agreement with the string states not only for the BPS states belonging to short multiplets but also for those belonging to intermediate multiplets as well. In fact, these intermediate multiplets provide a stronger test of the black-hole/string-state equivalence because the gyromagnetic ratios are not determined by supersymmetry alone, in contrast to those of the short multiplets.

In making this check, we shall make use of the following results in the literature: how to generate superpartners of extreme black holes using fermionic zero modes [10]; supersymmetric sum rules on magnetic moments [11, 12]; g -factors in heterotic string theory [13] and black hole solutions of heterotic strings on a torus [14, 15]. We will also need to bear in mind the following facts which contradict some popular beliefs:

(i) Although the classical value $g = 2$ is required in QED, the Standard Model and indeed open string theory [16], this is not a universal rule. Tree level unitarity applies only in the energy regime $M_{\text{pl}} > E > m/Q$ for a particle of mass m and charge Q [12]. (This also clears up an old paradox in Kaluza-Klein theory where the massive states have $g = 1$ [17, 18].) In the heterotic string we need two gyromagnetic ratios for left and right movers (g_L, g_R). This range is empty for graviphoton couplings and so there are no unitarity constraints on g_R ; and the condition $g_L = 2$ is required only for $N_L = 0$ states. They do, however, obey the sum rule⁵ $g_L + g_R = 2$ [13, 22].

(ii) Members of a supermultiplet do not necessarily have the same g -factor [11].

(iii) As we shall see in this paper, in extended supersymmetry, even members of a supermultiplet with the same spin do not necessarily have the same g -factor.

(iv) Rotating Kerr-Newman-Sen black holes do not have the same g -factor as an “electron” (or $N_L = 0$ string state); the former has $(g_L, g_R) = (0, 2)$ and the latter $(g_L, g_R) = (2, 0)$: the opposite way round!

(v) The superpartner of a non-rotating black hole is not a rotating black hole, since the

⁴Preliminary results from the present paper were presented by one of us (MJD) at *Strings 96*, Santa Barbara, July 1996 and the *European Network Meeting*, University of Crete, Heraklion, September 1996.

⁵Since the Type I and heterotic $SO(32)$ strings are related by strong/weak coupling [9, 19, 20, 21], it is tempting to suppose that this sum rule is related to the $g = 2$ condition for open strings.

former angular momentum is provided by the fermionic hair [10, 23] and the latter by the bosonic Kerr angular momentum.

(vi) In the *heterotic* string, extreme black holes are not necessarily supersymmetric, since both $M^2 = Q_R^2/8$ and $M^2 = Q_L^2/8$ are extreme but only the former is supersymmetric [3]. Indeed, as we shall also discuss here, it seems likely that the identification of string states with extreme black holes applies to non-BPS states as well.

(vi) In addition to magnetic dipole moments we also find electric dipole moments even for purely electrically charged black holes. The electric dipole moments of the corresponding string states have not yet been calculated directly but are consistent with heterotic/Type IIA duality.

Further dynamical evidence for the correspondence between strings and black holes was given in [24, 25] where comparisons were made between the low energy scattering amplitudes. The idea was then taken to address the microscopic origin of black hole entropy. It was shown [22, 26] that the entropy of these extremal black holes, evaluated at the stretched horizon [27], exactly matches the result expected from the degeneracy of string states (at least in the $N_L \gg 1$ limit). More recently, this analysis was put on more solid ground by focusing on dyonic states which have finite scalars on the horizon (*e.g.* the four- and five-dimensional Reissner-Nordström black holes). These states do not require the stretched horizon approach and their entropy is uniquely determined by their finite area of the horizon. However, it is slightly more subtle to count the number of associated string states since non-elementary excitations are involved. This problem could be solved [28, 29, 30] by analyzing the black holes in the Type II picture where the D-brane technology [31, 32] could be applied. As is by now well known, the results are in perfect agreement, giving further support to the conjecture that massive string states are described by extremal black holes [2].

It is well known that the low energy limit of the heterotic theory is described by a four dimensional $N = 4$ supergravity theory coupled to 22 vector multiplets. Denoting the vector and graviphoton electric charges by Q_L and Q_R respectively, the extremal electrically charged black holes preserving supersymmetry have masses given by

$$M^2 = \frac{1}{8} Q_R^2 \quad (1.1)$$

(where the dilaton VEV has been set to zero). Saturation of this Bogomol'nyi bound ensures that these black holes are not only supersymmetric, but also fall in the short representation of $N = 4$. On the other hand, the elementary string mass formula for the heterotic string is [33]

$$M^2 = \frac{1}{8} (Q_R^2 + 2(N_R - 1/2)) = \frac{1}{8} (Q_L^2 + 2(N_L - 1)). \quad (1.2)$$

Comparison of these expressions then indicates that if extremal black holes obeying (1.1) are to be identified with string states, they must correspond to states with oscillator numbers $N_R = 1/2$ and $N_L = 1 + (Q_R^2 - Q_L^2)/2$. Only for vanishing Kerr angular momentum, do solutions saturating this Bogomol'nyi bound exhibit an event horizon. Strictly speaking, therefore, the epithet *black holes* should be reserved only for the $L = 0$ solutions. However, in this paper we shall use the notion of black holes in a rather loose sense; we also refer to supersymmetric rotating states as black holes. Otherwise one might create the impression

that they are naked singularities whereas stringy corrections to the metric are expected and might well smooth out the singularity [34, 35].

Previous attempts at calculating gyromagnetic ratios of black holes have typically focused on the aspects of classically rotating solutions. Since all such black hole solutions to the low energy $N = 4$ theory have been constructed [14, 36, 15], it is then a simple matter of examining the asymptotic behavior of the gauge fields in order to read off the electric and magnetic dipole moments of any given solution. Based on this procedure, the supersymmetric electrically charged black holes were found to have gyromagnetic ratios $(g_L, g_R) = (0, 2)$ [14, 34] where, as in the definitions of Q_L and Q_R , g_L and g_R correspond to vector and graviphoton couplings respectively. This classical result may then be compared to the gyromagnetic ratios of elementary closed string states,

$$g_L = \frac{\langle S_R \rangle}{\langle S_R + S_L \rangle}, \quad g_R = \frac{\langle S_L \rangle}{\langle S_R + S_L \rangle}, \quad (1.3)$$

which were derived in [13]. In general the magnetic dipole moments are not strictly diagonal. Thus $\langle S_{R(L)} \rangle$ denotes the expectation value of the spin arising from the right(left)-handed sector of the string. Based on this comparison, it is then seen that the classical black hole solution corresponds to the $S_L \rightarrow \infty$ limit of the elementary heterotic string, which fits in well with the requirement that short representations of the string have $N_R = 1/2$ and hence must get their macroscopic spin from the left side of the string [14, 34].

Furthermore, the fact that $g_R = 2$ was taken as additional support for the idea of a black hole as a fundamental object [34]. However, as noted above, the standard argument for demanding $g = 2$, based on the requirement of tree level unitarity, need only apply in the energy range $M_{\text{pl}} > E > m/Q$ for a particle of mass m and charge Q [12]. Based on (1.2), it is easy to see that this range is essentially empty for graviphoton couplings so that there are no unitarity constraints on g_R . On the other hand, for g_L , we find that unitarity demands $g_L = 2$ for $N_L = 0$ states. Since these states contain the potentially massless gauge bosons responsible for symmetry enhancement at special points in the T^6 compactification, $g_L = 2$ (at least at tree level) also follows from the requirements of gauge invariance of the spontaneously broken Yang-Mills effective action. What this indicates is that, if anything, unitarity constraints give the opposite situation as found for the black holes, namely $(g_L, g_R) = (2, 0)$ for $N_L = 0$ states as opposed to $(g_L, g_R) = (0, 2)$ for black holes. Thus it is apparent that the truly microscopic string states are not well described by *classically* rotating black holes, even in the limiting case when the spin is taken to zero.

In order to understand the finite N_L states, it is necessary to realize that the superpartners of a black hole are not simply other classical solutions with bosonic Kerr-type angular momentum differing by small multiples of $1/2$. Instead, the additional spin of the black hole superpartners is provided by fermionic hair [10], so that a single bosonic solution along with its fermionic hair fills out a complete (in this case short) supermultiplet. The aim of the present work is to give a more complete description of supersymmetric black holes from a manifestly supersymmetric point of view, including both bosonic Kerr-like angular momentum and fermionic zero mode spin. In doing so, we are able to derive the gyromagnetic and gyroelectric ratios for all states in the black hole supermultiplet, and find complete agreement with the elementary heterotic string result, (1.3), for all values of N_L and not just in the limiting case. In the process we find in fact a much stronger result for short multiplets of

$N = 4$ supersymmetry—namely the gyromagnetic and gyroelectric ratios of the short multiplets are *completely* determined based on supersymmetry alone. It should be noted that, while it is already known that supersymmetry in general puts restrictions on the magnetic moments [11], unlike in that case, here there is no residual freedom remaining in the choice of transition moments in the short multiplets.

The situation is very different for black holes in intermediate multiplets. Always dyonic in the heterotic framework, a subclass is elementary in the Type II picture. Here we find that supersymmetry is far less restrictive. In fact, for fixed long-range fields (mass, charge and angular momentum) there exists a family of distinct supersymmetric black hole solutions [15]. Interestingly enough, they have different g -factors in heterotic variables. But after translation to the Type II language, all have the proper magnetic dipole moments of an elementary Type II string. Thus, we see that the analysis of gyromagnetic ratios is in fact quite powerful when it comes to intermediate multiplets. Besides the pleasing fact that there is an equivalence between Type II strings and extremal black holes we also learn that generically gyromagnetic ratios in intermediate multiplets are not purely determined by supersymmetry.

In the process of using supersymmetry to derive the magnetic dipole moments we discover the surprising fact that *elementary* string states can have electric dipole moments! That this is the case may be readily seen by examining elementary Kaluza-Klein states of the heterotic string and invoking string/string duality. Since these states have $Q_L = Q_R$ and $(g_L, g_R) = (2, 0)$ in the heterotic picture, they have a magnetic dipole moment in both the Kaluza-Klein and winding sectors (in order to give a purely left-handed combination). Under dualization, this state remains elementary, but based on the duality map must now have an electric dipole moment in the winding sector of the Type II string. In fact, an analysis reveals that already in the heterotic picture the fermionic members of this multiplet also have electric dipole moment. The complete scenario and interplay between gyromagnetic and gyroelectric ratios is consistent with string/string duality, agrees with the gyromagnetic ratios obtained from string theory, and is far more subtle than anticipated.

The paper is organized as follows. First, we review the low energy limit of the heterotic string compactified on a six-torus and some crucial facts of its duality to the Type II string compactified on $K3 \times T^2$. Since supersymmetry properties of the solutions are crucial in the analysis, we pay particular attention to the $N = 4$ supersymmetry transformations and briefly review the representation theory of massive $N = 4$ multiplets. In short, the supermultiplets are classified by their superspin L and the number of preserved supersymmetries q . Their structure was intensively discussed in [37] and [3]. Also, L was shown in [3] to correspond to the Kerr angular momentum of the supersymmetric black hole state without fermionic hair. In section three we first discuss general bosonic supersymmetric black holes. We then focus on states preserving two supersymmetries. By identifying their Killing spinors and fermionic zero modes we generate the full short supermultiplets (up to second order in the supersymmetry variations) and read off their electric and magnetic dipole moments. The particular example of a Kaluza-Klein state is analyzed in detail and the connection to the Type II picture is made. Then we discuss the short multiplets in more detail. In the heterotic language, those correspond (mainly) to purely electric (or purely magnetic) states.

In section four we emphasize some aspects of intermediate multiplets. The results for general states are derived and some comments on a bound state interpretation are made.

The freedom left by supersymmetry is emphasized. We then concentrate on states which are elementary in the Type II picture and compare their properties with Type II string states. A direct correspondence is then made between extremal supersymmetric black holes (with the appropriate quantum numbers) and elementary Type II string states, thus providing further evidence for the identification of black holes as elementary strings.

Before we conclude this paper we discuss the, perhaps surprising, possibility of interpreting some non-supersymmetric but nevertheless extremal black holes (like the one in [38]) as elementary heterotic string states with $N_L = 1$, $N_R = (Q_L^2 - Q_R^2 + 1)/2$. It is shown that the entropy, evaluated using the stretched horizon procedure, and the gyromagnetic ratios agree with those of non-BPS-saturated string states.

2 Four-dimensional $N = 4$ Supersymmetry

In this section we briefly review the low-energy action of the heterotic string compactified on T^6 and its duality to the Type IIA string compactified on $K3 \times T^2$. We restrict ourselves to the issues relevant for this paper, since this duality was historically the first one to be found and is widely discussed [8, 9, 39, 40]. Since representation theory lies at the heart of our results, we end this section with a summary of the massive $N = 4$ representations and how they may arise from the fermion zero-mode algebra.

2.1 The Heterotic Picture

Our starting point is the heterotic string compactified on a six-torus. The low-energy effective action for the string at a generic point on the Narain lattice is given by $N = 4$ supergravity with a total of 28 $U(1)$ gauge fields. However it is important to realize that, of these 28 gauge fields, six are singled out as graviphotons in the $N = 4$ theory, with different transformation properties from the other 22. In terms of $N = 4$ supersymmetry, the graviphotons fall in the graviton multiplet with field content

$$(g_{\mu\nu}, B_{\mu\nu}, A_\mu^{(R)a}, \eta, \psi_\mu^\alpha, \lambda^\alpha) , \quad (2.1)$$

where a and α are indices in the 6 and 4 dimensional representations respectively of the global $SU(4) \simeq SO(6)$ symmetry group of $N = 4$ supergravity. The other 22 $U(1)$ fields instead lie in 22 vector multiplets, given by

$$(A_\mu^{(L)I}, \varphi^{aI}, \chi^{\alpha I}) , \quad (2.2)$$

with I running from 1 to 22 labeling the multiplet.

The distinction between the graviphotons and the ordinary $U(1)$ fields also persists at the stringy level, with the six graviphotons originating from the right (supersymmetric) side of the heterotic string and the 22 vector fields from the left. For this reason we have used the labels (L) and (R) denoting the left- and right-sided world-sheet origins of the gauge fields. Due to the global $SU(4)$ invariance, all six graviphotons share identical properties. Similarly, from the supergravity point of view, all 22 vector multiplets may also be treated alike. Therefore the gyromagnetic ratios (and similarly gyroelectric ratios) for the 28 $U(1)$

fields are completely specified by just two numbers, g_L for the 22 vector multiplets and g_R for the graviphotons[14]. In the following, it will become obvious that the different transformation properties of the $N = 4$ graviton and vector multiplets gives rise to different values for g_L and g_R , in accord with their left- and right-sided origin on the string world sheet.

To be more precise about the labels (L) and (R) on the gauge fields, we give a brief review of the salient properties of $N = 4$ supergravity. For supergravity coupled to 22 vector multiplets, the 6×22 scalars φ^{aI} parametrize the coset $O(6, 22)/O(6) \times O(22)$ and may be written in terms of a vielbein, V , transforming as vectors of both $O(6, 22)$ and $O(6) \times O(22)$. The vielbein is a constrained 28×28 matrix satisfying

$$V^{-1} = LV^T \eta , \quad (2.3)$$

where L is the $O(6, 22)$ metric

$$L = \begin{pmatrix} 0 & I_6 & 0 \\ I_6 & 0 & 0 \\ 0 & 0 & -I_{22} \end{pmatrix} , \quad (2.4)$$

and

$$\eta = \begin{pmatrix} I_6 & 0 \\ 0 & -I_{22} \end{pmatrix} , \quad (2.5)$$

corresponding to the split of the vielbein into right- and left-sided [*i.e.* $O(6)$ and $O(22)$] components

$$V = \begin{bmatrix} V_R \\ V_L \end{bmatrix} . \quad (2.6)$$

Using the vielbein allows the construction of the $O(6, 22)$ matrix

$$\mathcal{M} = V^T V = V_L^T V_L + V_R^T V_R , \quad (2.7)$$

as well as the individual components

$$V_L^T V_L = \frac{1}{2}(\mathcal{M} - L) \quad V_R^T V_R = \frac{1}{2}(\mathcal{M} + L) . \quad (2.8)$$

Overall, there are 28 gauge fields A_μ with field strengths $F = dA$, both transforming as a vector of $O(6, 22)$. The vielbein is then used to translate these field strengths into left- and right-sided ones

$$\begin{aligned} F_{\mu\nu}^{(L)} &= V_L L F_{\mu\nu} \\ F_{\mu\nu}^{(R)} &= V_R L F_{\mu\nu} , \end{aligned} \quad (2.9)$$

corresponding to the $N = 4$ vectors and graviphotons respectively. Since the vielbein need not be constant in general, strictly speaking only the field strengths $F_{\mu\nu}$ and not the gauge fields A_μ themselves may be split into right- and left-sided parts. However, whenever the vielbein approaches its asymptotic value V^∞ , we may equally well refer to left- and right-sided gauge fields according to $A_\mu^{(R,L)} \simeq V_{R,L}^\infty L A_\mu$. This is sufficient for determining the

g -factors of the black holes since we only need to examine the asymptotic behavior of the fields at infinity.

In terms of the above definitions, the low-energy effective Lagrangian for the bosonic fields may be written in the compact form [41]

$$\begin{aligned} \mathcal{L} = & \frac{1}{16\pi G} \sqrt{-g} e^{-\eta} \left[R - \frac{1}{2} (\partial\eta)^2 - \frac{1}{12} e^{-2\eta} H^2 - \frac{1}{4} e^{-\eta} F_{\mu\nu}^{(R)T} F^{(R)\mu\nu} \right. \\ & \left. + \frac{1}{8} \text{Tr}(\partial\mathcal{M}L\partial\mathcal{M}L) - \frac{1}{4} e^{-\eta} F_{\mu\nu}^{(L)T} F^{(L)\mu\nu} \right] \end{aligned} \quad (2.10)$$

(using the canonical metric). The first line corresponds to the supergravity fields and the second line to the vector multiplets. For the heterotic string the 3-form H contains a Chern-Simons correction, $H = dB + \frac{1}{2} A^T L dA$. Note that the left- and right-sided field strengths may be combined to give

$$F_{\mu\nu}^{(R)T} F^{(R)\mu\nu} + F_{\mu\nu}^{(L)T} F^{(L)\mu\nu} = F_{\mu\nu}^T (L\mathcal{M}L) F^{\mu\nu} , \quad (2.11)$$

resulting in a more conventional form of the Lagrangian.

For non-rotating black hole solutions, the axion is not excited and may be consistently set to zero. However, for rotating black holes, the axion may not simply be discarded. Since we only examine black hole field configurations that solve the equations of motion, we take the liberty of dualizing the 3-form H according to

$$H_{\mu\nu\lambda} = e^{2\eta} \epsilon_{\mu\nu\lambda\sigma} \partial^\sigma a , \quad (2.12)$$

which defines the axion field a . This allows us to write the new Lagrangian

$$\mathcal{L}' = \frac{1}{16\pi G} \sqrt{-g} \left[R - \frac{|\partial S|^2}{2(S_2)^2} + \frac{1}{8} \text{Tr}(\partial\mathcal{M}L\partial\mathcal{M}L) - \frac{1}{4} S_2 F^T (L\mathcal{M}L) F + \frac{1}{4} S_1 F^T L * F \right] , \quad (2.13)$$

which gives rise to identical equations of motion. Here the dilaton and axion have been combined into the complex S field

$$S = a + i e^{-\eta} \quad (2.14)$$

The use of S results in a more compact structure in the subsequent analysis and is additionally more natural when considering the $SL(2; Z)_S$ properties of the black hole solutions. However nothing depends on this dualization, and we could equally well have retained the original 3-form H .

The bosonic equations of motion corresponding to either form of the $N = 4$ Lagrangian are given by [41]

$$\begin{aligned} R_{\mu\nu} &= \frac{\text{Re}(\partial_\mu S \partial_\nu \bar{S})}{2(S_2)^2} - \frac{1}{8} \text{Tr}(\partial_\mu \mathcal{M} L \partial_\nu \mathcal{M} L) + \frac{S_2}{2} (F_{\mu\alpha}^T L \mathcal{M} L F_\nu^\alpha - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}^T L \mathcal{M} L F^{\alpha\beta}) \\ \nabla_\mu (-S_2 \mathcal{M} L F^{\mu\nu} + S_1 * F^{\mu\nu}) &= 0 \\ \frac{\nabla^\mu \nabla_\mu S}{(S_2)^2} + i \frac{\partial_\mu S \partial^\mu \bar{S}}{(S_2)^3} + \frac{1}{4} F_{\mu\nu}^T L * F^{\mu\nu} - \frac{i}{4} F_{\mu\nu}^T L \mathcal{M} L F^{\mu\nu} &= 0 . \end{aligned} \quad (2.15)$$

These equations are invariant under both $O(6, 22; Z)_T$ and $SL(2; Z)_S$ dualities. The latter corresponds to the transformations

$$S \rightarrow \frac{aS + b}{cS + d} \quad \mathcal{F}_{\mu\nu} \rightarrow (cS + d)\mathcal{F}_{\mu\nu} , \quad (2.16)$$

($ad - bc = 1$) where the complex field strength $\mathcal{F}_{\mu\nu}$ is defined by

$$\mathcal{F}_{\mu\nu} = F_{\mu\nu} - i\mathcal{M}L * F_{\mu\nu} , \quad (2.17)$$

as well as non-trivial transformations on the fermionic fields.

In order to discuss the fermions and supersymmetry, we find it convenient to use a ten-dimensional notation for the $D = 4$, $N = 4$ spinors. Hence we introduce ten-dimensional gamma matrices with tangent space indices, Γ^A , $A = 0, 2, \dots, 9$ satisfying the Clifford algebra $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$. In the dimensional reduction to $D = 4$, each ten-dimensional Majorana-Weyl spinor decomposes into four $D = 4$ Majorana spinors and the gamma matrices split up into a set of four-dimensional and six-dimensional gamma matrices, given by γ^α , $\alpha = 0, 1, 2, 3$ and Γ^a , $a = 1, 2, \dots, 6$ respectively (a labels the 6 of $SU(4)$ as before). Curved space indices are then introduced in the usual manner, $\gamma^\mu = e^\mu_\alpha \gamma^\alpha$. We also define $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ (tangent space indices) so that $(\gamma^5)^2 = 1$.

For a purely bosonic background, the supersymmetry transformations of the fermions are given by

$$\begin{aligned} \delta\psi_\mu &= [\nabla_\mu - \frac{1}{4}i\gamma^5\frac{\partial_\mu S_1}{S_2} - \frac{1}{8\sqrt{2}}\sqrt{S_2}F_{\alpha\beta}^{(R)a}\gamma^{\alpha\beta}\gamma_\mu\Gamma^a + \frac{1}{4}Q_\mu^{ab}\Gamma^{ab}]\epsilon \\ \delta\lambda &= \frac{1}{4\sqrt{2}}[\gamma^\mu\frac{\partial_\mu(S_2 - i\gamma^5 S_1)}{S_2} - \frac{1}{2\sqrt{2}}\sqrt{S_2}F_{\mu\nu}^{(R)a}\gamma^{\mu\nu}\Gamma^a]\epsilon \\ \delta\chi &= \frac{1}{\sqrt{2}}[\gamma^\mu V_L L \partial_\mu V_R^T \cdot \Gamma - \frac{1}{2\sqrt{2}}\sqrt{S_2}F_{\mu\nu}^{(L)}\gamma^{\mu\nu}]\epsilon , \end{aligned} \quad (2.18)$$

where $Q_\mu^{ab} = (V_R L \partial_\mu V_R^T)^{ab}$ is the composite $SO(6)$ connection. The first two lines correspond to the gravitino and dilatino, which are both in the graviton multiplet, while the last line corresponds to the gaugino. We note that there is an obvious decoupling between the graviton and vector multiplets, with the graviphotons contributing to the former and the remaining 22 $U(1)$ field strengths contributing to the latter. Finally, we also need the lowest order supersymmetry variation of the metric and vector fields

$$\begin{aligned} \delta e_\mu^\alpha &= \frac{1}{2}\bar{\epsilon}\gamma^\alpha\psi_\mu \\ \delta A_\mu &= -\frac{1}{2}(S_2)^{-1/2}\bar{\epsilon}[\gamma_\mu V_L^T \chi - \sqrt{2}V_R^T \cdot \Gamma(\psi_\mu - \sqrt{2}\gamma_\mu\lambda)] . \end{aligned} \quad (2.19)$$

Although all 6 + 22 gauge fields transform together, the decoupling between graviphotons and vector multiplets is obvious in the above variation.

We now turn to the spectrum of this theory. This may be characterized by two central charges, Z_1 and Z_2 , which were found in [42, 36] to be

$$|Z_{1,2}|^2 = \frac{1}{8} \left[Q_R^2 + P_R^2 \pm 2 \left(Q_R^2 P_R^2 - (Q_R P_R)^2 \right)^{\frac{1}{2}} \right] , \quad (2.20)$$

where Q_R, P_R are the electric and magnetic charges of the six graviphotons with field strengths $F_{\mu\nu}^{(R)}$. For states without magnetic charge, this simply reduces to

$$|Z_1|^2 = |Z_2|^2 = \frac{1}{8}Q_R^2. \quad (2.21)$$

On the other hand, restricting to states carrying only charges in the first two graviphotons gives

$$\begin{aligned} |Z_1|^2 &= \frac{1}{8}[(Q_R^1 + P_R^2)^2 + (Q_R^2 - P_R^1)^2] \\ |Z_2|^2 &= \frac{1}{8}[(Q_R^1 - P_R^2)^2 + (Q_R^2 + P_R^1)^2]. \end{aligned} \quad (2.22)$$

This restriction simplifies the discussion and is useful when comparing heterotic states with Type II states which do not carry Ramond-Ramond charges.

The mass M of a solitonic state is bounded by the central charges through the Bogomol'nyi bound

$$M \geq \text{Max}\{Z_1, Z_2\}. \quad (2.23)$$

If $M = Z_1 = Z_2$ the state preserves two supersymmetries and is a member of a short supermultiplet. One preserved supersymmetry is indicated by $M = Z_1 > Z_2$ which puts the state into an intermediate multiplet. It is important that in $N = 4$ supergravity the masses of supersymmetric states are protected from quantum corrections. If the mass is larger than both central charges then one has a long supermultiplet, all supersymmetries are broken, and the mass is generically not protected from corrections. It is clear from (2.21) that elementary heterotic states are either short or long, but never intermediate.

Let us now turn to elementary states of the heterotic string. While the analysis is well known since the early days of string theory, we find it instructive to review a few facts. The heterotic string has non-supersymmetric oscillations from the left-handed sector denoted by $\tilde{\alpha}_{-n}^I$ and $\tilde{\alpha}_{-n}^\mu$. The right-handed sector is supersymmetric and has bosonic oscillators α_{-n}^a , α_{-n}^μ and fermionic ones b_{-r}^a and b_{-r}^μ acting on the Neveu-Schwarz vacuum (the Ramond sector may be treated similarly). The masses of the elementary string excitations are given by

$$\begin{aligned} M^2 &= \frac{1}{8} [Q_R^2 + 2(N_R - 1/2)] \\ &= \frac{1}{8} [Q_L^2 + 2(N_L - 1)] \end{aligned} \quad (2.24)$$

where the six Q_R and 22 Q_L are essentially superpositions of momentum and winding quantum numbers in the compactified directions (or Yang-Mills charges); they correspond precisely to the charges discussed before. The quantities N_R and N_L count the number of string oscillator excitations.

Comparing (2.24) with (2.23), one easily realizes, at least on the basis of masses and quantum numbers, that supersymmetric electrically charged states could potentially be identified with elementary string states with oscillator numbers

$$N_R = \frac{1}{2}, \quad N_L = \frac{1}{2}(Q_L^2 - Q_R^2) + 1. \quad (2.25)$$

All such states must necessarily lie in short multiplets, while $N_R > 1/2$ states fall in long multiplets. From the heterotic point of view no comparison can be made between properties of states in intermediate multiplets and elementary string states. However this is not the case for the dual Type II string, which does contain elementary intermediate states. Properties of this dual theory are summarized in the next subsection.

2.2 Duality to the Type II String

It is remarkable that the Type IIA string theory compactified on $K3 \times T^2$ gives an $N = 4$ theory with the exact same massless field content and moduli space as the heterotic string compactified on T^6 [43, 44]. In this case, however, the 28 $U(1)$ gauge fields arise as 24 Ramond-Ramond fields from $K3$ compactification as well as 2 Kaluza-Klein and 2 winding gauge fields from the T^2 . Of the six $N = 4$ graviphotons, four are Ramond-Ramond, and the remaining two are combinations of the T^2 fields. While much evidence has been provided in support of this duality, for the present work we only make use of the duality map given in [45, 42]. Details and conjectures based on this duality may be found in earlier publications [8, 9, 39, 40].

In order to examine the supersymmetries of the Type II picture, we note that the central charges derived from this theory are

$$|\tilde{Z}_{1,2}|^2 = \frac{1}{8} \left[\tilde{Q}^2 + \tilde{P}^2 \pm 2 \left(\tilde{Q}^2 \tilde{P}^2 - (\tilde{Q} \tilde{P})^2 \right)^{\frac{1}{2}} \right], \quad (2.26)$$

where \mathcal{Q} and \mathcal{P} are the charges of the six graviphotons expressed in the Type II picture. For details and notation see [42]. Comparing the central charges with those on the heterotic side, (2.20), indicates a straightforward mapping between the gauge fields in the two pictures. Avoiding the Ramond-Ramond fields, we restrict ourselves to gauge fields arising from the T^2 part of the compactification, and find

$$\begin{aligned} |\tilde{Z}_1|^2 &= \frac{1}{8} [(\tilde{Q}_R^1 + \tilde{P}_R^2)^2 + (\tilde{Q}_R^2 - \tilde{P}_R^1)^2] \\ |\tilde{Z}_2|^2 &= \frac{1}{8} [(\tilde{Q}_L^1 - \tilde{P}_L^2)^2 + (\tilde{Q}_L^2 + \tilde{P}_L^1)^2]. \end{aligned} \quad (2.27)$$

In general, the dictionary relating the heterotic and Type II fields is quite complicated. However, by focusing only on T^2 and assuming an essentially diagonal form of the asymptotic scalar matrix, the relevant part of the dictionary acts on the Kaluza-Klein and winding charges as

Heterotic	Type II	
Q_1	\tilde{Q}_1	
Q_2	\tilde{Q}_2	
Q_3	\tilde{P}_4	
Q_4	$-\tilde{P}_3$	

(2.28)

where 1,2 are Kaluza-Klein charges and 3,4 are winding charges. As seen here, dualization of the 3-form has the obvious effect of electric/magnetic duality on the winding gauge fields.

Left- and right-sided charges are defined, in both pictures, by

$$Q_{R,L}^i = \frac{1}{\sqrt{2}}(Q_i \pm Q_{i+2}) \quad (2.29)$$

and so on. Kaluza-Klein states, with only Q_1 and/or Q_2 excited, may clearly be elementary simultaneously in both points of view.

Let us now turn also to the perturbative spectrum of the Type II string. Since the Type II string carries supersymmetry on both sides we find for the mass formula [42]

$$\begin{aligned} M^2 &= \frac{1}{8}[(\tilde{Q}_R)^2 + 2(\tilde{N}_R - 1/2)] \\ &= \frac{1}{8}[(\tilde{Q}_L)^2 + 2(\tilde{N}_L - 1/2)] , \end{aligned} \quad (2.30)$$

From these formulas we learn by comparison with the central charges of (2.27) (with vanishing magnetic charges) that an elementary Type II string can be in short ($\tilde{N}_R = \tilde{N}_L = 1/2$), intermediate ($\tilde{N}_{R(L)} > \tilde{N}_{L(R)} = 1/2$) or long ($\tilde{N}_R, \tilde{N}_L > 1/2$) multiplets. This will enable us to compare the results for the gyromagnetic ratios of black holes in intermediate multiplets, which are out of reach of perturbative heterotic states, with elementary string states.

2.3 Massive Supermultiplets and $N = 4$ Representation Theory

Before studying magnetic moments and gyromagnetic ratios, one must first understand the nature of spinning black holes. General rotating black holes carrying both electric and magnetic charge have been constructed by solving the bosonic equations of motion⁶ with vanishing background fermions [14, 36, 15]. Some properties of these solutions were outlined above. In addition to their mass, angular momentum and 28 dimensional electric and magnetic charge vectors, the black holes also have electric and magnetic dipole moments. Due to the dichotomy between graviphotons and vector multiplets, the dipole moments are specified by left- and right-sided gyromagnetic and gyroelectric ratios.

In order to relate these rotating black holes to elementary string states, it is important to realize that, since the low-energy theory is described by $N = 4$ supergravity, all black holes must fall into irreducible $N = 4$ representations, whether long, intermediate or short ones. We recall that massive representations of supersymmetry may be labeled by their mass M and superspin L (in addition to the central charges). A complete representation is then built up by taking the $2L + 1$ states of superspin L and acting on each of them with an appropriate combination of supercharges. For the long representation of $N = 4$, all supercharges are active, giving rise to a $(2L + 1) \times 2^{2N} = (2L + 1) \times (128 + 128)$ dimensional multiplet. Since the supercharges are evenly distributed between helicities $\pm \frac{1}{2}$, we see that generically (for $L \geq N/2$) the long representation has spins running from $L - N/2$ to $L + N/2$ (*i.e.* from $L - 2$ to $L + 2$). For $L < N/2$ the smallest spin is always 0. The multiplet structure is given in Table 1.

For states saturating a Bogomol'nyi bound, however, not all of the supercharges are active, as some are annihilated by the specific representation. In this case the dimensions of

⁶In actuality it is often sufficient to start from a known classical background, *e.g.* the Kerr solution, and use various symmetries to generate a complete family of solutions [14].

Spin	$L = 0$	$L = \frac{1}{2}$	$L = 1$	$L = 3/2$	$L = 2$	Spin	L
4					1	$L + 2$	1
$\frac{7}{2}$				1	8	$L + \frac{3}{2}$	8
3			1	8	28	$L + 1$	28
$\frac{5}{2}$		1	8	28	56	$L + \frac{1}{2}$	56
2	1	8	28	56	70	L	70
$\frac{3}{2}$	8	28	56	70	56	$L - \frac{1}{2}$	56
1	27	56	70	56	28	$L - 1$	28
$\frac{1}{2}$	48	69	56	28	8	$L - \frac{3}{2}$	8
0	42	48	27	8	1	$L - 2$	1

Table 1: Massive $N = 4$ long representations

Spin	$L = 0$	$L = \frac{1}{2}$	$L = 1$	Spin	L
2			1	$L + 1$	1
$\frac{3}{2}$		1	4	$L + \frac{1}{2}$	4
1	1	4	6	L	6
$\frac{1}{2}$	4	6	4	$L - \frac{1}{2}$	4
0	5	4	1	$L - 1$	1

Table 2: Massive $N = 4$ short representations

the representations become much smaller. In particular, for the short $N = 4$ representation preserving half of the supersymmetries, only half of the supercharges are active, yielding a $(2L + 1) \times 2^N = (2L + 1) \times (8 + 8)$ dimensional multiplet. The spin content for a short multiplet ranges from $L - 1$ to $L + 1$ (for $L \geq 1$) and is summarized in Table 2 along with the exceptional cases. Finally, states preserving a quarter of the supersymmetries (Table 3) fall in intermediate multiplets of dimension $(2L + 1) \times 2^{3N/2} = (2L + 1) \times (32 + 32)$. However they are not present in the elementary heterotic string spectrum as they must carry non-zero magnetic charge.

Given this digression into supersymmetry representation theory, it should now be apparent that the Kerr angular momentum of the bosonic solutions does not correspond directly to the spin of the state, but instead to its superspin L . While the bosonic solution is just a single member of a supermultiplet, all its superpartners may be generated by the action of the fermion zero modes, corresponding to building up a complete representation using the supercharges⁷. This approach was taken in Ref. [10] to construct exact superpartners

⁷This shows in particular that all superpartners of the bosonic black hole solution have non-trivial fermionic backgrounds and allows the possibility of spinning superpartners to the extremal black holes

Spin	$L = 0$	$L = \frac{1}{2}$	$L = 1$	$L = 3/2$	Spin	L
3				1	$L + \frac{3}{2}$	1
$\frac{5}{2}$			1	6	$L + 1$	6
2		1	6	15	$L + \frac{1}{2}$	15
$\frac{3}{2}$	1	6	15	20	L	20
1	6	15	20	15	$L - \frac{1}{2}$	15
$\frac{1}{2}$	14	20	15	6	$L - 1$	6
0	14	14	6	1	$L - \frac{3}{2}$	1

Table 3: Massive $N = 4$ intermediate representations

to the bosonic solution in the context of short $N = 2$ representations. The method was also applied in [46, 47] where further properties of $N = 2$ black holes were studied. In the following, we extend this to short and intermediate $N = 4$ representations in the context of supersymmetric elementary black hole solutions.

Previous discussions of black hole gyromagnetic ratios have generally ignored the fermion zero modes, and have instead focused on the bosonic solutions with a given Kerr angular momentum. Since the different members of a supermultiplet may have different g factors⁸, this yields an incomplete description of the gyromagnetic ratios when $N = 4$ supersymmetry is taken into account. Furthermore, since Kerr angular momentum L corresponds directly to superspin, it is not possible to compute the g factors for the different spin components of a supermultiplet solely by varying L . Consideration of the fermion zero modes is unavoidable if one wishes to study the properties of all superpartners to the black holes.

In order to examine a complete black hole supermultiplet, we start with a purely bosonic solution and then build up the rest of the representation by acting on it with the fermionic zero modes. Denoting the purely bosonic solution by Φ , the remaining members of the supermultiplet are then encoded in the finite transformation [10]

$$\Phi \rightarrow e^{\delta} \Phi = \Phi + \delta \Phi + \frac{1}{2} \delta \delta \Phi + \dots . \quad (2.31)$$

Since the supersymmetry transformation δ is given in terms of Grassman parameters, the finite transformation eventually terminates. However, even for the short $N = 4$ representation, this has terms up to order δ^8 . So unlike the case for short $N = 2$ representations, it is unrealistic to expect to derive the exact superpartners in the present case. We thus work only up to the second order term since that is sufficient to see the corrections to the bosonic background. In fact, the transformation rules confirm this claim, since fermionic transformations always introduce a derivative of a bosonic field (or other fermions if one was to consider higher order fermionic terms). Since the bosonic fields for black hole solutions generically behave like $B \sim a + b/r$, second order corrections to the bosonic fields are of order

without naked singularities.

⁸While the g factors for different members of a supermultiplet are in general distinct, they are however not completely independent, but must instead satisfy general sum rules [11].

$O(\frac{1}{r^2})$ which is precisely what we need for magnetic and electric dipole moments. Higher order supersymmetry variations induce modifications of order $O(\frac{1}{r^4})$ or higher and hence do not modify the results derived from second order variations.

3 Black Holes and String States in Short Multiplets

After summarizing a few known results concerning bosonic supersymmetric black holes and massive supermultiplets, we proceed to examine the electric and magnetic dipole moments of states in short multiplets. It turns out that supersymmetry alone is sufficient to determine the properties of the short multiplets. Nevertheless the appearance of electric dipole moments, while necessitated by supersymmetry, is somewhat unexpected from the elementary string point of view. We present a careful examination of the electric dipole moments in the case of the Kaluza-Klein black hole, which may be studied from both the heterotic and Type II side.

3.1 Supersymmetric Black Holes

A general black hole solution may be given in terms of its bosonic fields, solving the classical equations of motion, (2.15). It is then possible to read off the properties of this solution from the asymptotic behavior of the fields at infinity. In order to incorporate Kerr angular momentum it is necessary to use a parametrization of the space-time metric that includes off-diagonal g_{0i} components. We find it convenient to start with a vierbein of the form

$$e_\mu^\alpha = \begin{pmatrix} e^A & 0 \\ e^A C_i & e^{-A} \hat{e}_i^a \end{pmatrix}, \quad (3.1)$$

so that the metric is

$$g_{\mu\nu} = \begin{pmatrix} -e^{2A} & -e^{2A} C_j \\ -e^{2A} C_j & e^{-2A} \hat{g}_{ij} - e^{2A} C_i C_j \end{pmatrix}. \quad (3.2)$$

While this decomposition is completely general, the e^A factors are motivated by the correspondence to the rotating black hole metrics of [14, 15]. Asymptotic behavior of this metric determines both the mass and the angular momentum of the black hole. Using the relations

$$\begin{aligned} g_{00} &\sim -\left(1 - \frac{2M}{r}\right) \\ g_{0i} &\sim 2\epsilon_{ijk} L_j \frac{\hat{x}_k}{r^2} \end{aligned} \quad (3.3)$$

then gives

$$\begin{aligned} \partial_i A &\sim M \frac{\hat{x}_i}{r^2} \\ C_i &\sim -2\epsilon_{ijk} L_j \frac{\hat{x}_k}{r^2} \end{aligned} \quad (3.4)$$

for the metric (3.2), assuming $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$ at infinity.

For electric and magnetic charges, we use the definitions

$$\begin{aligned} E_i &\equiv F_{0i} \sim Q \frac{\hat{x}_i}{r^2} \\ B_i &\equiv \frac{1}{2} \epsilon_{ijk} F_{jk} \sim P \frac{\hat{x}_i}{r^2}. \end{aligned} \quad (3.5)$$

Since these charges are governed by the leading behavior of the fields, the asymptotic values of the moduli, V^∞ , are sufficient to map between the left and right sided charges and the combined 28 $U(1)$ charges via $Q_{L,R} = V_{L,R}^\infty LQ$. The electric (\vec{d}) and magnetic ($\vec{\mu}$) dipole moments are readily obtained from the gauge fields A_μ themselves according to

$$\begin{aligned} A_0 &\sim d_i \frac{\hat{x}_i}{r^2} \\ A_i &\sim \epsilon_{ijk} \mu_j \frac{\hat{x}_k}{r^2}. \end{aligned} \quad (3.6)$$

By now, the bosonic rotating black hole solutions of toroidally compactified string theory are completely known [14, 48, 36, 49, 15]. These solutions have been constructed by acting with particular $O(8, 24)$ transformations on the Kerr solution. A subclass of solutions which is general enough to investigate dyonic black holes was studied in [15]. These states are characterized by 5 parameters: m, l, α, β and γ . m and l correspond to the mass and the rotation parameter of the original Kerr solution. The three angles α, β and γ are $O(8, 24)$ boost parameters which generate the solution of interest. In general, such a solution gives a non-supersymmetric black hole. Supersymmetric black holes are obtained generically by setting $m \rightarrow 0$ while (at least some of) the angles approach infinity. The condition for the absence of naked singularities is

$$|l| \leq m \quad (3.7)$$

which implies that supersymmetric black holes have either no angular momentum or admit naked singularities. As mentioned in the introduction we do mind the presence of such singularities, since stringy corrections will certainly enter and change the short-range space-time structure anyway.

Based on the asymptotic behavior of the solution, the essential properties of the black holes are given by [15, 36]⁹

$$\begin{aligned} M &= \frac{1}{2} m (\cosh \beta^2 + \cosh \alpha \cosh \gamma) \\ L &= \frac{1}{2} l m \cosh \beta (\cosh \alpha + \cosh \gamma) \\ Q_R^a &= -\sqrt{2} m \sinh \gamma \cosh \alpha \delta_{a,1} \\ Q_L^I &= -\sqrt{2} m \cosh \gamma \sinh \alpha \delta_{I,1} \\ P_R^a &= -\sqrt{2} m \cosh \beta \sinh \beta \delta_{a,2} \\ P_L^I &= \sqrt{2} m \cosh \beta \sinh \beta \delta_{I,2} \\ d_R^a &= \sqrt{2} l m \cosh \alpha \sinh \beta \delta_{a,2} \end{aligned} \quad (3.8)$$

⁹Note that some of the signs have been changed to match our charge and dipole moment conventions.

$$\begin{aligned}
d_L^I &= -\sqrt{2}lm \cosh \gamma \sinh \beta \delta_{I,2} \\
\mu_R^a &= -\sqrt{2}lm \sinh \gamma \cosh \beta \delta_{a,1} \\
\mu_L^I &= -\sqrt{2}lm \sinh \alpha \cosh \beta \delta_{I,1} .
\end{aligned}$$

Note that use of appropriate $O(6, 22)$ rotations allows non-trivial charge configurations to be generated from this solution, which only excites two left- and two right-sided gauge fields. Comparing the above charge and mass formulas with (2.20), we can extract the supersymmetric states in the appropriate limits. To have a purely electrically charged supersymmetric bosonic black hole, we have to take the limit $(m \rightarrow 0; \gamma \rightarrow \infty)$ or $(m \rightarrow 0; \alpha, \gamma \rightarrow \infty)$ with M constant. Here, we notice already the subtle issue of limits. For example, taking $\alpha = \gamma \rightarrow \infty$ leaves us with the same mass and charge configuration but different gyromagnetic ratios compared with a two-step procedure of first sending γ and then α to infinity¹⁰. Even worse is the situation for intermediate states, which are dyonic in this heterotic language. Consider the states which are elementary in the Type II language, i.e. have $Q_R = Q_L$ and $P_R = -P_L$. For those, β , α and γ have to diverge; however the details of the procedure are crucial for the electric and magnetic moments. This will prove to be important later.

3.2 Gyromagnetic Ratios of Short Supermultiplets

The starting point for the fermion zero mode construction is the general rotating black hole solution. In the absence of fermionic hair, this black hole is completely specified by its mass M and Kerr angular momentum \vec{L} , read off from the asymptotic behavior of the metric.

In general, rotating black hole solutions are quite complicated. However, in comparing the $a = \sqrt{3}$ and $a = 1$ black holes to elementary string states, since these states preserve exactly half of the supersymmetries, this fact alone automatically determines many of the important properties of these black holes. Therefore we start, not with the general equations of motion, (2.15), but instead with the Killing spinor equations derived from the supersymmetry variations, (2.18). Since the graviton and vector multiplets have different properties, we examine them individually. Specializing to the graviton multiplet, we see that in order for a non-trivial background to admit a Killing spinor, the various terms in both the gravitino and dilatino variations must balance each other out. As a consequence, for backgrounds preserving partial supersymmetry, the graviphotons (and hence their charges) must be related to the dilaton-axion field.

For short multiplets preserving exactly half of the supersymmetries, we seek a black hole ansatz where a chiral half of the supersymmetry transformations parametrized by ϵ is projected out. In order to determine the form of this projection, we first specialize to a non-rotating electric black hole. In this case anticipate a solution with a diagonal metric and vanishing axion and magnetic fields. The dilatino variation of (2.18) then reduces to

$$\delta\lambda = \frac{1}{4\sqrt{2}}\gamma^i \left[\frac{\partial_i S_2}{S_2} + \frac{1}{\sqrt{2}}\sqrt{S_2} E_i^{(R)a} \gamma^0 \Gamma^a \right] \epsilon , \tag{3.9}$$

¹⁰This result is not necessarily in conflict with the no-hair theorem, as the gyromagnetic ratios only have meaning for rotating solutions, which are actually naked singularities in this limit.

so that preserving half of the supersymmetries demands the relation

$$\frac{1}{\sqrt{2}}\sqrt{S_2}E_i^{(R)a} = \hat{n}^a\sqrt{-g_{00}}\frac{\partial_i S_2}{S_2}, \quad (3.10)$$

in which case (3.9) reduces to

$$\delta\lambda = \frac{1}{2\sqrt{2}}\gamma^i\frac{\partial_i S_2}{S_2}P_{\hat{n}}\epsilon. \quad (3.11)$$

Since there are six graviphotons, \hat{n} is a unit vector selecting which combination of graviphotons is excited. This variation vanishes for exactly half of the supersymmetries since $P_{\hat{n}}$ is a projection operator,

$$P_{\hat{n}} = \frac{1}{2}(1 + \gamma^{\bar{0}}\hat{n} \cdot \Gamma) \quad (3.12)$$

($\gamma^{\bar{0}}$ denotes the Dirac matrix with flat space index). A complete solution is then obtained by demanding that the gravitino (as well as gaugino) variation also vanishes under the *identical* projection.

Using the projection $P_{\hat{n}}$ as the basis for constructing Bogomol'nyi saturated electric black holes, we now give the complete rotating solution. Choosing the parametrization of the metric given in (3.2) and demanding that half of the supersymmetry is preserved according to the projection given by (3.12) now results in the first order equations

$$\begin{aligned} E_i^{(R)} &= -\sqrt{2}\hat{n}\partial_i\left(\frac{1}{S_2}\right) \\ B_i^{(R)} &= \sqrt{2}\hat{n}\epsilon_{ijk}\partial_j\left(\frac{C_k}{S_2}\right), \end{aligned} \quad (3.13)$$

for the graviphotons and

$$\begin{aligned} E_i^{(L)}\hat{n} &= -\sqrt{2}\frac{1}{S_2}V_L L\partial_i V_R^T \\ B_i^{(L)} &= -\epsilon_{ijk}E_j^{(L)}C_k \end{aligned} \quad (3.14)$$

for the left-sided $U(1)$ fields, as well as the conditions

$$\begin{aligned} e^{-2A} &= S_2 \\ \partial_i S_1 &= -\frac{\hat{g}_{ij}}{\sqrt{\hat{g}}}\epsilon_{jkl}\partial_k C_l \\ Q_i^{ab} &= 0, \end{aligned} \quad (3.15)$$

and the requirement that \hat{g}_{ij} is a flat metric¹¹. For simplicity we have assumed that the dilaton VEV vanishes at infinity. However it is easy to verify that the gyromagnetic ratios are independent of the dilaton VEV, as both charges and dipole moments scale similarly.

¹¹For a non-rotating solution these equations are a special case of the Killing spinor equations given in [36] where the more general case of dyonic BPS saturated black holes preserving both 1/2 and 1/4 of the supersymmetries was considered. In this subsection we only focus on states preserving exactly 1/2 of the supersymmetries, as only they may be identified as elementary heterotic string states.

Remarkably enough, the above conditions already demonstrate the consistency of the identification of black holes and elementary string states. In the case of $Q_R = Q_L$ the string states are required to have $N_L = 1$. Therefore the angular momentum arising from string oscillators on the left-hand side is essentially zero. From the black hole point of view this constraint was somewhat mysterious, although addressed in higher dimensions in [34]. Here we see a direct four-dimensional origin of it. In the case of $Q_R = Q_L$ it turns out that the black hole solutions have a vanishing axion. But since the axion is tied to the angular momentum by supersymmetry, as given in (3.15), it forces L to vanish! This is a very pleasing correspondence between both pictures.

Using the above ansatz preserving half of the supersymmetries, the variations of the fermionic fields may be expressed as

$$\begin{aligned}
\delta\psi_0 &= \frac{1}{2}\hat{\gamma}^i \frac{\partial_i(S_2 - i\gamma^5 S_1)}{S_2} (S_2)^{-1} \gamma_{\bar{0}} P_{\hat{n}} \epsilon \\
\delta\psi_i &= -\frac{1}{2}\hat{\gamma}^j \frac{\partial_j(S_2 - i\gamma^5 S_1)}{S_2} (\hat{\gamma}_i - (S_2)^{-1} C_i \gamma_{\bar{0}}) P_{\hat{n}} \epsilon \\
\delta\lambda &= \frac{1}{2\sqrt{2}} \hat{\gamma}^i \frac{\partial_i(S_2 - i\gamma^5 S_1)}{S_2} (S_2)^{-1/2} P_{\hat{n}} \epsilon \\
\delta\chi &= -\gamma^{\bar{0}} \hat{\gamma}^i E_i^{(L)} (S_2)^{1/2} P_{\hat{n}} \epsilon ,
\end{aligned} \tag{3.16}$$

provided $\epsilon = (S_2)^{-1/4} \epsilon_0$ with ϵ_0 constant. These equations vanish for Killing spinors ϵ_+ satisfying $P_{\hat{n}} \epsilon_+ = 0$. The remaining spinors,

$$\epsilon_- \quad \text{such that} \quad P_{\hat{n}} \epsilon_- = \epsilon_- \tag{3.17}$$

then give rise to the fermion zero modes used to generate the complete supermultiplet. We note that this ansatz clearly shows the difference between the six graviphotons (part of the graviton multiplet) versus the 22 vector multiplets. This will come out explicitly in the results for the left- and right-sided gyromagnetic ratios and electric dipole moments.

It is important to realize that the conditions (3.13), (3.14) and (3.15) for preserving half of the supersymmetries are necessary but not sufficient to ensure that the bosonic background is indeed a good black hole solution. Recall that in the low-energy supergravity theory, proof of the Bogomol'nyi bound through Nester's procedure is valid only on-shell. Equivalently we only have an on-shell formulation of $N = 4$ supergravity. Thus the bosonic equations of motion, (2.15), must still be imposed on the black hole solutions. Nevertheless, this ansatz for electric black holes, based only on the requirement of partially unbroken supersymmetry, is sufficient to determine many properties of the rotating black holes without having to refer to the equations of motion. As a result, this indicates that supersymmetric black holes derive many of their characteristics (including gyromagnetic and gyroelectric ratios) solely as a consequence of their supersymmetry. Taking the supersymmetric limit of the general rotating solution given above (and constructed by Sen for the case of pure electric charges [14, 22, 50]), we verify that the supersymmetry ansatz is indeed satisfied. Similarly, in the purely electric limit, the dyonic BPS saturated black holes of [36] preserve half of the supersymmetries and manifestly satisfy the Killing spinor equations by construction.

Before incorporating the fermion zero modes, we first read off the properties of the bosonic background based on the electric black hole ansatz (3.12). From the asymptotic form of the

metric, (3.13) shows that the right-handed electric charge vector satisfies

$$Q_R = -2\sqrt{2}M\hat{n} , \quad (3.18)$$

which is precisely the statement that the short multiplet saturates the Bogomol'nyi bound, given this choice of normalization factors. For the magnetic field, (3.13) is the statement that the right-sided magnetic dipole moment is related to the angular momentum:

$$\begin{aligned} \vec{\mu}_R &= -2\sqrt{2}\vec{L}\hat{n} \\ &= \frac{Q_R}{M}\vec{L} . \end{aligned} \quad (3.19)$$

On the other hand, (3.14) indicates that the left-sided electric fields are related to the scalar components of the vector multiplets and not to any part of the graviton multiplet. Additionally, the left-sided magnetic dipole moment vanishes, $\vec{\mu}_L = 0$, as both the left-sided electric field and the Kerr angular momentum fall off as $1/r^2$.

Defining the gyromagnetic ratio by

$$\vec{\mu} = \frac{gQ}{2M}\vec{J} \quad (3.20)$$

gives the result

$$g_L = 0, \quad g_R = 2 , \quad (3.21)$$

in agreement with previous results for a black hole carrying Kerr angular momentum [14, 34]. At this point we wish to reinforce the notion that this result is completely general for any classically rotating solution generated by the $N = 4$ short ansatz (3.12), and is derived from supersymmetry properties alone, independent of any specific form of the black hole solution or the equations of motion. In order to complete this result for general members of the $N = 4$ short representation, we must now consider the action of the fermion zero modes on this bosonic background.

Given the bosonic solution constructed to satisfy the Killing spinor equations, all that remains is to generate its superpartners by incorporating the fermion zero modes. In particular, since we wish to examine the gyromagnetic ratios, we must study the effect of the zero modes on both the metric (for the angular momentum) and the $U(1)$ fields (for the electric and magnetic dipole moments). Since we are only interested in the first non-trivial order for the fermions, it is sufficient to look at terms to lowest order in the fermion fields in the supersymmetry variations. The relevant transformations are those of the vierbein and the vector fields, given in (2.19).

The variation of g_{0i} yields a quantum correction to the angular momentum. Since the gravitino transformation for the electric black hole is given by (3.16), we find

$$\delta\delta g_{0i} = -\frac{1}{2}\bar{\epsilon}_- \left[\frac{\partial_j S_2}{S_2} \frac{\hat{g}_{im}}{\sqrt{\hat{g}}} \epsilon_{mjk} \hat{\gamma}_k i \gamma^5 - C_i \frac{\partial_j S_1}{(S_2)^2} \hat{\gamma}^j i \gamma^5 \right] (S_2)^{-1/2} \epsilon_- \quad (3.22)$$

where ϵ_- is a zero mode spinor defined by (3.17). While this expression is valid everywhere, we are only interested in its asymptotic form. In this case, since $\partial_i S_1 \sim 1/r^3$ and $C_i \sim 1/r^2$

are subdominant, the second term in (3.22) may be dropped, so the asymptotic behavior is given by

$$\begin{aligned}\delta\delta g_{0i} &\sim -\frac{1}{2}\epsilon_{ijk}\frac{\partial_j S_2}{S_2}(\bar{\epsilon}_-\hat{\gamma}^k i\gamma^5\epsilon_-) \\ &\sim -\epsilon_{ijk}M\mathcal{C}^j\frac{\hat{x}^k}{r^2},\end{aligned}\tag{3.23}$$

where $\mathcal{C}^i = (\bar{\epsilon}_-\hat{\gamma}^i i\gamma^5\epsilon_-)$ and we have used the relation between the asymptotic value of the dilaton and the ADM mass. This indicates that, in addition to the Kerr angular momentum of the original rotating black hole solution, the supersymmetry transformations generate quantum spin in direct correspondence with their representation theory (where we view the fermion zero modes ϵ_- as creation and annihilation operators). The total angular momentum thus takes the form

$$\vec{J} = \vec{L} + \vec{S},\tag{3.24}$$

where

$$\vec{S} = -\frac{M}{2}\vec{\mathcal{C}}\tag{3.25}$$

is the spin generated by the fermion zero modes and \vec{L} is the Kerr angular momentum (corresponding to superspin) of the bosonic solution.

For the variation of the vector fields, as given in (2.19), we note that both left- and right-sided gauge fields are combined into the 28 dimensional $O(6, 22)$ vector A_μ . Nevertheless, the form of δA_μ shows the relation between the left-sided gauge fields and the gauginos [transforming as a vector of $O(22)$] and the relation between the right-sided gauge fields (graviphotons) and the gravitino and dilatino. The magnetic dipole moment is generated from the spatial components of the gauge fields

$$\delta\delta A_i = \frac{1}{2}\bar{\epsilon}_-\left[i\gamma^5 V_L^T E_j^{(L)}\frac{\hat{g}_{im}}{\sqrt{\hat{g}}}\epsilon_{mjk}\hat{\gamma}_k - (\gamma^{\bar{0}}V_L^T E_i^{(L)} + V_R^T \cdot \Gamma\hat{n}^T \cdot E_i^{(R)})\right](S_2)^{1/2}\epsilon_-.\tag{3.26}$$

Asymptotically, as the vielbein approaches V^∞ , the second term becomes pure gauge. In this case only the first term is important, and it is immediately apparent that it induces a zero mode correction to the magnetic dipole moment. Substituting in the charges at infinity, we find

$$\delta\delta A_i \sim \frac{1}{2}\epsilon_{ijk}V_L^T Q_L \mathcal{C}^j\frac{\hat{x}^k}{r^2},\tag{3.27}$$

so that $\delta\delta\mu_i = \frac{1}{2}V_L^T Q_L \mathcal{C}_i$. Transforming to left- and right-handed fields according to (2.9) and using the definition of spin given by (3.25), we find

$$\delta\delta\vec{\mu}_L = \frac{Q_L}{M}\vec{S} \quad \delta\delta\vec{\mu}_R = 0.\tag{3.28}$$

Previously we have seen that the purely bosonic BPS saturated electric black hole has a magnetic moment on the right, but not the left side. However it is clear from supersymmetry that spinning up the black holes with the fermion zero modes creates instead a magnetic moment only on the left. Putting both results together, we see that

$$\vec{\mu}_L = \frac{Q_L}{M}\vec{S} \quad \vec{\mu}_R = \frac{Q_R}{M}\vec{L},\tag{3.29}$$

Spin	Multiplicity		
	1	4	5
$L + 1$	$\frac{2}{L+1}$		
$L + \frac{1}{2}$		$\frac{1}{L+1/2}$	
L	$\frac{2}{L(L+1)}$		0
$L - \frac{1}{2}$		$-\frac{1}{L+1/2}$	
$L - 1$	$-\frac{2}{L}$		

Table 4: Left-sided gyromagnetic ratios (g_L) for short multiplets. The right-sided (graviphoton) gyromagnetic ratios are given by $g_R = 2 - g_L$.

where the total angular momentum is given by $\vec{J} = \vec{L} + \vec{S}$. We note that in discussing the complete superpartners, we must appeal to a semi-classical argument in order to make a direct correspondence with supersymmetry representation theory since here we are adding quantum mechanical spin \vec{S} to classical Kerr angular momentum \vec{L} . In this case, we see that the magnetic moments do not in general commute with total spin \vec{J} , leading to transition dipole moments as well as diagonal ones. Noting that the gyromagnetic ratios defined in (3.20) apply only to diagonal magnetic moments, we pick out the z -axis as a preferred direction so that the g factors may be written as

$$g_L = \frac{2\langle S^z \rangle}{L^z + S^z} \quad g_R = \frac{2\langle L^z \rangle}{L^z + S^z} . \quad (3.30)$$

These gyromagnetic ratios are summarized in Table 4.

Although derived for electric black holes in $N = 4$ supergravity, the gyromagnetic ratios that we have found are identical to those of elementary string states [13, 14], even up to the identification of the Kerr angular momentum L with the left side and the supermultiplet generating spin S with the right side of the heterotic string. Therefore this fits nicely with the identification of black holes with fundamental string states. However, it should be clear in the above derivation that supersymmetry alone has guaranteed this result. Thus one may argue that this has not yet provided a serious test of black holes as string states. In the next section, however, when considering intermediate states, we find that supersymmetry in itself is no longer sufficient to fully constrain the electric and magnetic dipole moments. Hence in this case a comparison of the gyromagnetic and gyroelectric ratios will indeed provided a meaningful test of this conjecture.

3.3 Electric dipole moments for superpartners

We now turn to an examination of the electric dipole moments of the supersymmetric black holes. For the electrically charged solution described above, it is curious that, unlike for the magnetic dipole moments, supersymmetry alone is insufficient to determine the electric dipole moments of the purely bosonic state. This may be seen by noting that non-zero electric

dipole moments may be balanced against somewhat more complicated scalar asymptotics in Eqns. (3.13) and (3.14) of the supersymmetry conditions. In fact non-rotating supersymmetric black holes have even been constructed (using a different method) that nevertheless have an electric dipole moment [51]. The occurrence of these seemingly surprising quantities can also be understood from a slightly different angle. Essentially, supersymmetry and the equations of motion require (at least for $L = 0$) $e^{-\eta}$, e^{-2A} and some other fields to be harmonic functions. The canonical choice is a one-center solution without higher moments. However, one can also add them, leading to electric dipole moments, without violating supersymmetry or the equations of motion. This results in solutions which are not spherically symmetric and which have a singularity structure worse than the standard solution¹². Hence, we will not consider those kinds of solutions here. Further evidence for disregarding them is provided by the Kaluza-Klein black hole in the Type II picture, which will be discussed later. The main point is that an electric dipole moment in the heterotic language would be inconsistent with Type II stringy considerations.

On the other hand, the fermion zero modes give an additional electric dipole moment contribution to the superpartners which is completely determined by supersymmetry. For the zero modes we find

$$\delta\delta A_0 = \frac{1}{2}\bar{\epsilon}_- \left[\hat{n} \cdot E_i^{(R)} \hat{\gamma}^i \gamma_{\bar{0}} V_R^T \cdot \Gamma \right] (S_2)^{-1/2} \epsilon_- , \quad (3.31)$$

leading to a contribution to the right-sided electric dipole moment

$$\delta\delta \vec{d}_R^a = -\frac{1}{2} \hat{n} \cdot Q_R \vec{\mathcal{D}}^a , \quad (3.32)$$

where $\mathcal{D}^{ia} = (\bar{\epsilon}_- \gamma_{\bar{0}} \hat{\gamma}^i \Gamma^a \epsilon_-)$.

Unlike for the zero-mode-induced magnetic dipole moment, $\delta\delta \vec{\mu}_L$, the resulting electric dipole moment is not proportional to the zero-mode spin \vec{S} . Furthermore, an electric dipole moment is generated for potentially all six graviphotons, even though the electric charge is carried by a single combination proportional to \hat{n} . Noting that $\hat{n} \cdot \vec{\mathcal{D}} = 0$, it is in fact only the five graviphotons *orthogonal* to \hat{n} that couple to the electric dipole moment. In other words, when an electric charge is turned on under a given graviphoton, it never picks up an electric dipole moment, while the other five do.

In general, since $\vec{\mathcal{D}}^a$ and $\vec{\mathcal{C}}$ have a different structure, the zero-mode electric dipole operator does not commute with spin, leading to transition moments and the impossibility of diagonalizing all five graviphoton electric dipole moments simultaneously. For a particular graviphoton (orthogonal to \vec{n}), it is possible to see that $\vec{\mathcal{D}} = \pm \vec{\mathcal{C}}$, where the sign depends on the chirality of the zero mode spinor ϵ_- in a specific six-dimensional subspace of the original ten-dimensional space-time. In particular, it is instructive to consider the T^2 truncation of the toroidally compactified heterotic string. Picking $\hat{n}^a = \delta^{a1}$ then gives

$$\delta\delta \vec{d}_R^1 = 0$$

¹²The reader might argue that the singularity issue is rather delicate in general; for example all solutions with $L \neq 0$ admit naked singularities. We conjecture that there are “good” and “bad” naked singularities, depending on whether they can be removed by stringy effects or not. Since electric dipole moments in non-rotating solutions make the space-time more singular, it appears natural to classify those configurations as “bad” ones.

$$\delta\delta\vec{d}_R^2 = \pm \frac{Q_R}{M} \vec{S}, \quad (3.33)$$

keeping in mind that $\delta\delta\vec{d}_R^a$ for $a = 3, 4, 5, 6$ are non-vanishing (and not diagonal) in this basis.

Since supersymmetry alone does not fix the electric dipole moments of a short black hole, we now turn to the explicit solution of [14]. This rotating electric black hole corresponds to setting $\beta = 0$ in the more general dyonic solution with charges given by Eqn. (3.8). In this case it is easy to see that the bosonic state has vanishing electric dipole moments. Furthermore, examination of (3.33) indicates that only the four spin $L + 1/2$ and the four spin $L - 1/2$ members of the superspin L short multiplet have electric dipole moments (if we start out with a solution without an electric dipole moment). This also demonstrates that even potentially light states, such as the spin $1/2$ “gauginos” of the $L = 0$ multiplet have electric dipole moments. While on the surface this appears troublesome for low-energy phenomenology, it should be noted that such electric dipole moments appear only in the graviphoton couplings, and not to the ordinary $U(1)$ vector fields.

In retrospect, the appearance of electric dipole moments is not really surprising from the point of view of heterotic/Type II duality. Focusing on T^2 , we consider an electric Kaluza-Klein state which may be elementary in both pictures. From the heterotic point of view we let this state carry electric charge $Q_1 = q$ (*i.e.* $Q = (q, 0, 0, 0)$ where the first two components are Kaluza-Klein charges and the last two are winding charges), so that $Q_L^1 = Q_R^1 = q/\sqrt{2}$. Focusing on the $L = 0$ case (since $N_L = 1$, the only possibilities are $L = 0$ or $L = 1$), we have seen that the heterotic gyromagnetic ratios are given by $(g_L, g_R) = (2, 0)$. In turn, this implies that $(\vec{\mu}_L^1, \vec{\mu}_R^1) = (-2\sqrt{2}\vec{J}, 0)$, or, in terms of Kaluza-Klein and winding fields, $\vec{\mu} = (-2\vec{J}, 0, 2\vec{J}, 0)$. Using the duality map of (2.28), this translates into the Type II side as

$$\begin{aligned} \vec{\mu}_1 &= -2\vec{J} \\ \vec{d}_4 &= -2\vec{J}, \end{aligned} \quad (3.34)$$

so that, starting only with magnetic dipole moments, we are inevitably led to consider electric dipole moments.

To complete the picture we may also use the duality map in reverse, and start with an elementary Type II string. Since the stringy formula, (1.3), is in fact applicable to *all* closed strings, it enables us to determine the gyromagnetic ratios of the Kaluza-Klein state in the Type II picture. Based on an orbifold construction of the $K3$ compactification, the short superspin 0 multiplet is generated by the left/right spin combination $[(1/2)+2(0)]_L \times [(1/2)+2(0)]_R = [(1) + 4(1/2) + 5(0)]$, and has gyromagnetic ratios

$$(\tilde{g}_L, \tilde{g}_R) = \begin{bmatrix} (1, 1) \\ 2 \times (2, 0), 2 \times (0, 2) \\ 5 \times (\cdot, \cdot) \end{bmatrix}, \quad (3.35)$$

in agreement with the $L = 0$ Kaluza-Klein black hole results given in Table 5¹³. Translating these magnetic moments to the heterotic picture then requires that electric dipole moments are present in the heterotic side as well.

¹³Note that in particular one finds vanishing magnetic dipole moments for the $S = 0$ members, which implies that the electric dipole moments should vanish in the heterotic picture, as discussed before.

spin		Heterotic				Type II			
		F_1	F_2	F_3	F_4	\tilde{F}_1	\tilde{F}_2	\tilde{F}_3	\tilde{F}_4
	Q	q	0	0	0	q	0	0	0
(1)	$\vec{\mu}$	$-2\vec{J}$	0	$2\vec{J}$	0	$-2\vec{J}$	0	0	0
	\vec{d}	0	0	0	0	0	0	0	$-2\vec{J}$
$2 \times (1/2)$	$\vec{\mu}$	$-2\vec{J}$	0	$2\vec{J}$	0	$-2\vec{J}$	0	$2\vec{J}$	0
	\vec{d}	0	$-2\vec{J}$	0	$-2\vec{J}$	0	$-2\vec{J}$	0	$-2\vec{J}$
$2 \times (1/2)$	$\vec{\mu}$	$-2\vec{J}$	0	$2\vec{J}$	0	$-2\vec{J}$	0	$-2\vec{J}$	0
	\vec{d}	0	$2\vec{J}$	0	$2\vec{J}$	0	$2\vec{J}$	0	$-2\vec{J}$
$5 \times (0)$	$\vec{\mu}$	0	0	0	0	0	0	0	0
	\vec{d}	0	0	0	0	0	0	0	0

Table 5: Electric charges and magnetic and electric dipole moments of the $L = 0$ Kaluza-Klein black hole supermultiplet in both the heterotic and Type II pictures.

What we have basically shown is that supersymmetry itself is sufficient to require the existence of electric dipole moments in the graviphoton couplings of short superspin L multiplets (although it does not necessarily fix their values, as demonstrated in [51]). It remains an open issue how these electric dipole moments originate from the string world sheet point of view. Since such electric dipole moments are presumably not present in the bosonic string, we conjecture that they arise as a consequence of world-sheet supersymmetry, and perhaps only in the Ramond sector of the superstring. This picture is of course consistent with the states shown in Table 5.

4 Intermediate Multiplets and Gyromagnetic Ratios

While we have seen that supersymmetry leaves no freedom for the electric and dipole moments of states in short multiplets, it turns out that the states in intermediate multiplets are far less constrained. By using a combination of “electric” and “magnetic” projection operators in the Killing spinor equations, we are able to extend the short multiplet black hole ansatz of the previous section to cover the case of intermediate multiplets. In doing so, we find the intriguing picture that a dyonic black hole may essentially be viewed as a combination of separate electrically and magnetically charged black holes.

Although the heterotic string has no elementary states in intermediate multiplets, we may use the duality map to go over to the Type II picture where such states do exist. Thus we make a comparison of certain intermediate black holes with elementary Type II string states. The resulting picture is considerably more complicated than that for short multiplets. At the end of this section we also make a few comments on how this analysis may be extended to consider non-supersymmetric states.

4.1 Supersymmetry and the intermediate multiplet solution

We start by recalling that many properties of the basic supersymmetric electric black hole solution may be determined starting from the electric projection operator $P_{\hat{n}}$ of Eqn. (3.12). Since the $N = 4$ supersymmetries and equations of motion are S -duality invariant, it is clear that dyonic black holes in *short* multiplets may similarly be constructed by means of a duality rotated projection,

$$P_{\hat{n}}^{\theta} = \frac{1}{2}(1 + e^{i\gamma^5\theta}\gamma^{\bar{0}}\hat{n} \cdot \Gamma) . \quad (4.1)$$

In particular, choosing $\theta = \pi/2$ corresponds to a magnetic projection, and leads to short magnetic black holes.

Intermediate states, on the other hand, may be constructed using a combination of the electric projection operator $P_{\hat{n}}$ and a magnetic projection

$$\tilde{P}_{\hat{m}} = \frac{1}{2}(1 + i\gamma^5\gamma^{\bar{0}}\hat{m} \cdot \Gamma) . \quad (4.2)$$

We note that such intermediate states are always dyonic in the heterotic language since no duality rotation may remove completely the magnetic charge generated by the combination of $P_{\hat{n}}$ and $\tilde{P}_{\hat{m}}$.

As in the previous section, construction of the intermediate solution starts from the general rotating metric ansatz, (3.2). However this time we demand that only a quarter of the supersymmetries are preserved. In this case, the expressions are more complicated and have more freedom. The resulting graviphoton fields are given by

$$\begin{aligned} E_i^{(R)} &= -\sqrt{2}e^{A+\eta/2}(\hat{n}\partial_i(A+\eta/2) - \hat{m}\partial_i X) \\ \frac{\hat{g}_{ij}}{\sqrt{\hat{g}}}(B_j^{(R)} + \epsilon_{jkl}E_k^{(R)}C_l) &= \sqrt{2}e^{-(A-\eta/2)}(\hat{m}\partial_i(A-\eta/2) + \hat{n}\partial_i Y) , \end{aligned} \quad (4.3)$$

where \hat{n} and \hat{m} determine the graviphoton electric and magnetic charge vectors respectively. The angular momentum, specified by C_i , has been split into two contributions,

$$e^{2A}\frac{\hat{g}_{ij}}{\sqrt{\hat{g}}}\epsilon_{jkl}\partial_k C_l = \partial_i X + \partial_i Y , \quad (4.4)$$

where

$$\partial_i Y \equiv \partial_i X - e^{\eta}\partial_i a . \quad (4.5)$$

As a result, (4.3) indicates that both electric and magnetic graviphoton dipole moments are induced by non-vanishing Kerr angular momentum, in proportion to possible splitting of angular momentum between $\partial_i X$ and $\partial_i Y$. At this point it is convenient to make the asymptotic definitions of masses,

$$\begin{aligned} \partial_i(A+\eta/2) &\sim 2M_1\frac{\hat{x}_i}{r^2} \\ \partial_i(A-\eta/2) &\sim 2M_2\frac{\hat{x}_i}{r^2} , \end{aligned} \quad (4.6)$$

and angular momenta,

$$\begin{aligned}\partial_i Y &\sim 2\partial_i \left(\frac{\vec{L}_1 \cdot \hat{x}}{r^2} \right) \\ \partial_i X &\sim 2\partial_i \left(\frac{\vec{L}_2 \cdot \hat{x}}{r^2} \right),\end{aligned}\tag{4.7}$$

so that the intermediate black hole has mass $M = M_1 + M_2$ and Kerr angular momentum $\vec{L} = \vec{L}_1 + \vec{L}_2$. While this splitting may appear *ad hoc*, as we will see, it has its basis in the “splitting” of the dyonic black hole into individual electric and magnetic components.

It is crucial that the functions X and Y are determined not only by the Kerr angular momentum (which characterizes the superspin of the multiplet), but also by the axion, which is off-hand unrelated to the supersymmetry representation. The conclusion is that states with the same spin L and the same charges could have a different splitting of their spin into L_1 and L_2 components, which ultimately leads to different g -factors. Thus the case of intermediate multiplets is already quite different from that of short multiplets. In fact, this ambiguity is precisely reflected by the limit dependence in (3.8). The difference between α and γ does not affect the charges, the mass and the angular momentum, as long those angles are eventually taken to be infinite. However the moments are very sensitive to the particular choice of how the limit is obtained. In short, supersymmetry allows for a wide class of solutions with the same quantum numbers, but different dipole moments and hence different gyromagnetic/gyroelectric ratios. We will see later how string states tie in with this. But it is already apparent that supersymmetry in general no longer fixes the gyromagnetic ratios of states in intermediate representations.

For the left-sided gauge fields, the tale is somewhat more complicated. Since these fields belong to $N = 4$ vector multiplets, the condition for generating intermediate states now relates the scalar and vector components of this multiplet according to

$$\sqrt{2}V_L L \partial_i V_R^T = -e^{-(A+\eta/2)} E_i^{(L)} \hat{n} - e^{A-\eta/2} \frac{\hat{g}_{ij}}{\sqrt{\hat{g}}} (B_j^{(L)} + \epsilon_{jkl} E_k^{(L)} C_l) \hat{m} .\tag{4.8}$$

Due to the presence of both electric and magnetic terms, unlike for the short ansatz, here no general statement can be made about the left-sided electric and magnetic dipole moments. Only after solving the equations of motion can one examine either the asymptotics of the scalars or the left-sided gauge fields to determine what dipole moments are generated. So once again we find considerably more freedom for intermediate multiplets.

With this intermediate state ansatz, the Killing spinor equations following from (2.18) become

$$\begin{aligned}\delta\psi_0 &= -\frac{1}{2}e^{2A}\hat{\gamma}^i\gamma_{\bar{0}}[\partial_i(A+\eta/2+i\gamma^5Y)P_{\hat{n}}+\partial_i(A-\eta/2+i\gamma^5X)\tilde{P}_{\hat{m}}]\epsilon \\ \delta\psi_i &= \frac{1}{2}\hat{\gamma}^j(\hat{\gamma}_i-e^{2A}C_i\gamma_{\bar{0}})[\partial_j(A+\eta/2+i\gamma^5Y)P_{\hat{n}}+\partial_j(A-\eta/2+i\gamma^5X)\tilde{P}_{\hat{m}}]\epsilon \\ \delta\lambda &= -\frac{1}{2\sqrt{2}}e^A\hat{\gamma}^i[\partial_i(A+\eta/2-i\gamma^5Y)P_{\hat{n}}-\partial_i(A-\eta/2-i\gamma^5X)\tilde{P}_{\hat{m}}]\epsilon \\ \delta\chi &= -e^A\gamma_{\bar{0}}\hat{\gamma}^i[e^{-(A+\eta/2)}E_i^{(L)}P_{\hat{n}}-i\gamma^5e^{A-\eta/2}\frac{\hat{g}_{ij}}{\sqrt{\hat{g}}}(B_j^{(L)}+\epsilon_{jkl}E_k^{(L)}C_l)\tilde{P}_{\hat{m}}]\epsilon.\end{aligned}\tag{4.9}$$

In addition to the gauge field ansatz, the composite $SO(6)$ connection vanishes, $Q^{ab} = 0$, and the spinor parameter is related to a constant spinor ϵ_0 according to $\epsilon = e^{(A+i\gamma^5 X)/2}\epsilon_0$. In general, these expressions decompose as a sum of two sets of conditions, one based on $P_{\hat{n}}$ and the other based on $\tilde{P}_{\hat{m}}$. In order to construct an intermediate state preserving a quarter of the supersymmetries, it is easiest to impose $\hat{n} \cdot \hat{m} = 0$ so that both projections commute with each other. This condition is essentially a no-force condition ensuring that the electric and magnetic black hole states generated by $P_{\hat{n}}$ and $\tilde{P}_{\hat{m}}$ are orthogonal.

In fact, based on this construction, it is apparent that the dyonic black holes preserving a quarter of the supersymmetries may be viewed as a combination of an electric and a magnetic black hole. Generating complete supermultiplets via the fermion zero modes, we find that the electrically charged state has mass $M_1 = -\hat{n} \cdot Q_R / 2\sqrt{2}$ and left- and right-sided magnetic dipole moments given by

$$(\vec{\mu}_L, \vec{\mu}_R) = (\vec{\mu}_L^{(0)}, 0) + \frac{1}{M_1}(Q_L \vec{S}_1, Q_R \vec{L}_1) , \quad (4.10)$$

where $\vec{\mu}_L^{(0)}$ indicates the original left-sided magnetic moment which is undetermined by supersymmetry¹⁴. As before, \vec{L}_1 and \vec{S}_1 correspond to Kerr angular momentum and quantum spin, respectively, with

$$\vec{S}_1 = -\frac{M_1}{2}(\bar{\epsilon}\gamma^i\gamma^5 P_{\hat{n}}\epsilon) . \quad (4.11)$$

Additionally, the fermion zero modes generate graviphoton electric dipole moments for the five graviphotons orthogonal to \hat{n}

$$\vec{d}_R^a = -\frac{1}{2}\hat{n} \cdot Q_R(\bar{\epsilon}\gamma_{\bar{0}}\gamma^a P_{\hat{n}}\epsilon) . \quad (4.12)$$

The magnetically charged state, on the other hand, has mass $M_2 = \hat{m} \cdot P_R / 2\sqrt{2}$ and electric dipole moments

$$(\vec{d}_L, \vec{d}_R) = (\vec{d}_L^{(0)}, 0) - \frac{1}{M_2}(P_L \vec{S}_2, P_R \vec{L}_2) , \quad (4.13)$$

this time with

$$\vec{S}_2 = -\frac{M_2}{2}(\bar{\epsilon}\gamma^i\gamma^5 \tilde{P}_{\hat{m}}\epsilon) . \quad (4.14)$$

Graviphoton magnetic dipole moments are also generated according to

$$\vec{\mu}_R^a = \frac{1}{2}\hat{m} \cdot P_R(\bar{\epsilon}\gamma_{\bar{0}}\gamma^a \tilde{P}_{\hat{m}}\epsilon) . \quad (4.15)$$

The complete dyon then gets its properties from a sum of the individual electric and magnetic contributions. In particular, the mass $M = M_1 + M_2 = (-\hat{n} \cdot Q_R + \hat{m} \cdot P_R) / 2\sqrt{2}$ confirms that this dyon saturates the Bogomol'nyi bound appropriate for an intermediate $N = 4$ state. This description of intermediate black holes is summarized in Table 6.

¹⁴Since $\vec{\mu}_L^{(0)}$ is not fixed by supersymmetry, at this point there is no way to assign it to either the electric or the magnetic state. In the next subsection, we see that in fact $\vec{\mu}_L^{(0)}$ belongs to the *magnetic* state, and not the electrically charged one.

	Electric state	Magnetic state
Projection	$P_{\hat{n}} = \frac{1}{2}(1 + \gamma^0 \hat{n} \cdot \Gamma)$	$\tilde{P}_{\hat{m}} = \frac{1}{2}(1 + i\gamma^5 \gamma^0 \hat{m} \cdot \Gamma)$
Masses	M_1	M_2
Kerr angular momentum	\vec{L}_1	\vec{L}_2
Charges	$Q_R = -2\sqrt{2}M_1\hat{n}$ $Q_L = \dots$	$P_R = 2\sqrt{2}M_2\hat{m}$ $P_L = \dots$
Dipole moments	$\vec{\mu}_R = -2\sqrt{2}\vec{L}_1\hat{n}$ $\vec{\mu}_L = \dots$	$\vec{d}_R = -2\sqrt{2}\vec{L}_2\hat{m}$ $\vec{d}_L = \dots$
Zero-mode spin	$\vec{S}_1 = -\frac{1}{2}M_1(\bar{\epsilon}\vec{\gamma}i\gamma^5 P_{\hat{n}}\epsilon)$	$\vec{S}_2 = -\frac{1}{2}M_2(\bar{\epsilon}\vec{\gamma}i\gamma^5 \tilde{P}_{\hat{m}}\epsilon)$
Zero-mode generated dipole moments	$\delta\delta\vec{\mu}_L = \frac{Q_L}{M_1}\vec{S}_1$ $\delta\delta\vec{d}_R = \pm \frac{\hat{n}\cdot Q_R}{M_1}\vec{S}_1$	$\delta\delta\vec{d}_L = -\frac{P_L}{M_2}\vec{S}_2$ $\delta\delta\vec{\mu}_R = \pm \frac{\hat{m}\cdot P_R}{M_2}\vec{S}_2$

Table 6: Properties of intermediate states, viewed as a combination of individual electric and magnetic states. Note that the splitting of angular momentum as well as some left-sided dipole moments are undetermined by supersymmetry. The \pm sign for the zero-mode-generated graviphoton dipole moments is only schematic, and is discussed in further detail in the text.

We must emphasize that, while there is a natural decomposition of the intermediate black hole into its “electric” and “magnetic” parts, there is only a single set of fermion zero modes which act on both parts simultaneously. Thus it is not necessarily the case that we may conclude from this ansatz that the intermediate black hole is a direct result of combining two marginally stable short black holes. Any true bound state interpretation of such dyonic black holes certainly requires a more careful analysis of this solution.

4.2 Black Holes and Type II Strings

It is now very interesting to compare those intermediate black hole states with elementary string states. Since the elementary heterotic string admits no intermediate states, this comparison can only be made to the Type II string, which does have elementary excitations in the intermediate multiplets. Since the general picture is somewhat complicated, we first consider the large superspin limit, $L \gg 1$, which corresponds to the classically rotating black hole where only Kerr angular momentum needs to be considered. After examining the purely bosonic solution, we then study the incorporation of fermion zero modes to generated the complete intermediate multiplet.

In the $L \gg 1$ limit the string mass formula, (2.30), requires either $\tilde{N}_R = 1/2$ or $\tilde{N}_L = 1/2$ for states in an intermediate multiplet. Thus the Kerr angular momentum must come entirely from either left- or right-sided string oscillators. Using the string formula for gyromagnetic ratios, (1.3), leads to

$$\tilde{g} = (0, 2) \quad \text{or} \quad \tilde{g} = (2, 0) , \quad (4.16)$$

for $\tilde{N}_R = 1/2$ or $\tilde{N}_L = 1/2$ states respectively.

This may now be compared with the bosonic black hole solutions of [14, 36, 15], with asymptotic behavior given in Eqn. (3.8). Intermediate dyonic states may be generated by taking the simultaneous limit $\alpha, \beta, \gamma \rightarrow \infty$ and $m \rightarrow 0$ while keeping the following quantities fixed:

$$\begin{aligned} M_1 &= \frac{1}{2}m \cosh \alpha \cosh \gamma \\ M_2 &= \frac{1}{2}m \cosh^2 \beta \\ L_1 &= \frac{1}{2}lm \cosh \beta \cosh \gamma \\ L_2 &= \frac{1}{2}lm \cosh \beta \cosh \alpha . \end{aligned} \tag{4.17}$$

The resulting supersymmetric interpretation is then that of Table 6, with

$$\begin{aligned} Q_R = Q_L &= -2\sqrt{2}M_1 \\ \mu_R = d_L &= -2\sqrt{2}L_1 \end{aligned} \tag{4.18}$$

for the electric state, and

$$\begin{aligned} P_R = -P_L &= -2\sqrt{2}M_2 \\ d_R = -\mu_L &= 2\sqrt{2}L_2 \end{aligned} \tag{4.19}$$

for the magnetic state. Note that by looking at an explicit solution, the left-sided dipole moments are completely determined. Furthermore, we see that in fact μ_L and d_L are more naturally connected to the magnetic and electric pieces respectively of the intermediate state ansatz.

The heterotic fields may now be translated to the Type II picture using the duality dictionary given in Eqn. (2.28). The resulting Type II charges are given in Table 7, where $\Delta L \equiv L_1 - L_2$. Examination of the table indicates a surprising and interesting result: despite the freedom in taking the supersymmetric limit of (3.8), expressed by the conditions (4.17) (or, equivalently, the supersymmetry ambiguity expressed in the functions X and Y), this arbitrariness completely vanishes from the magnetic dipole moments in the Type II picture. In fact, for this black hole, we find $\tilde{Q}_R = -2\sqrt{2}M$, hence saturating the Bogomol'nyi bound for $\tilde{N}_R = 1/2$. The resulting magnetic moment is purely right-sided, with $\tilde{\mu}_R = -2\sqrt{2}L = (\tilde{Q}_R/M)L$, leading to the gyromagnetic ratios $\tilde{g} = (0, 2)$, in agreement with the elementary Type II string picture. Since the X and Y ambiguity resides solely in the Type II electric dipole moments, we again anticipate a string calculation of electric dipole moments to complete the picture. However, since $\tilde{d}_L = -2\sqrt{2}\Delta L$, this suggests $\Delta L \approx 0$ if the assumption that such electric dipole moments originate from the Ramond sector of the string is to be correct. In turn this indicates the necessity of setting $\alpha = \gamma$ in Eqn. (3.8), which ensures that magnetic charges in the Type II picture vanish identically before taking the limit.

To complete the picture for intermediate multiplets, we must now consider the fermion zero modes. Starting in the heterotic picture, we recall from (4.18) and (4.19) that the charges of the black hole are

$$\begin{aligned} Q_R = Q_L &= -2\sqrt{2}M_1 \\ P_R = -P_L &= -2\sqrt{2}M_2 . \end{aligned} \tag{4.20}$$

	Heterotic				Type II			
	F_1	F_2	F_3	F_4	\tilde{F}_1	\tilde{F}_2	\tilde{F}_3	\tilde{F}_4
Q	q	0	0	0	q	0	p	0
P	0	0	0	p	0	0	0	0
μ	$-2L$	0	$-2\Delta L$	0	$-2L$	0	$-2L$	0
d	0	$-2\Delta L$	0	$2L$	0	$-2\Delta L$	0	$2\Delta L$

Table 7: Charges and dipole moments of the dyonic black hole in the $L \gg 1$ limit. Note that q and p have the same sign.

Translated to the Type II picture, this corresponds to a purely electric state with

$$\begin{aligned}\tilde{Q}_R &= -2\sqrt{2}M \\ \tilde{Q}_L &= -2\sqrt{2}(M_1 - M_2) .\end{aligned}\tag{4.21}$$

The basic zero mode algebra for the intermediate state is somewhat more complicated than that for short multiplets. Given the general spinor ϵ , a quarter of the components correspond to Killing spinors. The remaining components are split as one quarter for the electric state (\vec{S}_1), one quarter for the magnetic state (\vec{S}_2), and finally one quarter active for both. Note that, depending on M_1 and M_2 , the zero modes in general have to be normalized independently, so that the total spin, $\vec{S} = \vec{S}_1 + \vec{S}_2$, is appropriately quantized in basic units of $1/2$. Denoting the representation generating creation operators (built out of the zero modes) as $Q_1^\dagger, Q_2^\dagger, \dots, Q_6^\dagger$, and focusing on spin along the z -axis, we find the quantities listed in Table 8, where $\xi = M_1/(M_1 + M_2)$ represents the splitting of mass M into M_1 and M_2 . The Type II dipole moments have been calculated using the map

$$\begin{aligned}\tilde{\mu}_{L,R} &= \frac{1}{2}[(\mu_R + \mu_L) \pm (d_R - d_L)] \\ \tilde{d}_{L,R} &= \frac{1}{2}[\pm(\mu_R - \mu_L) + (d_R + d_L)] ,\end{aligned}\tag{4.22}$$

which follows from Eqn. (2.28).

In the last line of Table 8 we have listed the gyromagnetic ratios corresponding to the particular zero mode. This is only meant to be schematic, as in general several zero modes may be simultaneously active, and furthermore the dipole moments need to be combined with those of the bosonic state, given in Table 7. Nevertheless, we see that the non-overlapping operators, $Q_1^\dagger, Q_2^\dagger, Q_5^\dagger$, and Q_6^\dagger all have $\tilde{g} = (0, 2)$, while the overlapping operators have $\tilde{g} = (2, 0)$, hinting at a left-right splitting on the Type II world sheet. Avoiding the complication of adding Kerr angular momentum to the zero-mode-generated angular momentum, we now specialize to the superspin zero multiplet. In this case it is not hard to see that the four creation operators corresponding to $\tilde{g} = (0, 2)$ generate 16 states with spins $[(1) + 4(\frac{1}{2}) + 5(0)]$, while the other two generate 4 states with spins $[(\frac{1}{2}) + 2(0)]$. This corresponds *precisely* to the expected gyromagnetic ratios of the $L = 0$ intermediate multiplet generated from the elementary Type II string, with $\tilde{N}_R = 1/2$, and the left-right splitting of spins on the worldsheet, $[(1) + 4(\frac{1}{2}) + 5(0)]_L \times [(\frac{1}{2}) + 2(0)]_R$.

	Q_1^\dagger	Q_2^\dagger	Q_3^\dagger	Q_4^\dagger	Q_5^\dagger	Q_6^\dagger
S_1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}\xi$	$-\frac{1}{2}\xi$	0	0
S_2	0	0	$\frac{1}{2}(1-\xi)$	$-\frac{1}{2}(1-\xi)$	$\frac{1}{2}$	$-\frac{1}{2}$
$S = S_1 + S_2$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
μ_L	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}\xi$	$\sqrt{2}\xi$	0	0
μ_R	0	0	$\sqrt{2}(1-\xi)$	$-\sqrt{2}(1-\xi)$	$-\sqrt{2}$	$\sqrt{2}$
d_L	0	0	$-\sqrt{2}(1-\xi)$	$\sqrt{2}(1-\xi)$	$-\sqrt{2}$	$\sqrt{2}$
d_R	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}\xi$	$\sqrt{2}\xi$	0	0
$\tilde{\mu}_L$	0	0	$\sqrt{2}(1-2\xi)$	$-\sqrt{2}(1-2\xi)$	0	0
$\tilde{\mu}_R$	$-\sqrt{2}$	$\sqrt{2}$	0	0	$-\sqrt{2}$	$\sqrt{2}$
\tilde{d}_L	$\sqrt{2}$	$-\sqrt{2}$	0	0	$-\sqrt{2}$	$\sqrt{2}$
\tilde{d}_R	0	0	$-\sqrt{2}$	$\sqrt{2}$	0	0
$(\tilde{g}_L, \tilde{g}_R)$	(0, 2)	(0, 2)	(2, 0)	(2, 0)	(0, 2)	(0, 2)

Table 8: Intermediate state generating creation operators and their properties in both the heterotic and Type II pictures. Of the six operators, Q_1^\dagger , Q_3^\dagger and Q_5^\dagger raise the spin by $1/2$, and Q_2^\dagger , Q_4^\dagger and Q_6^\dagger lower the spin by $1/2$. Note that Q_3^\dagger and Q_4^\dagger have special significance as zero modes that are active under both electric and magnetic projections.

We have now seen that the gyromagnetic ratios of black holes do indeed correspond in the expected manner to that of elementary Type II states. Namely for $\tilde{N}_R = 1/2$ states, the Kerr angular momentum is generated from the left-side of the Type II string, giving $\tilde{g} = (0, 2)$, whereas the fermion zero modes filling out the intermediate representation correspond to both left- and right-sided spin on the Type II worldsheet. Thus we are left with a most pleasing conclusion. On one side, the black hole solutions under investigation have the properties of elementary Type II strings. On the other hand, this result was not at all enforced by supersymmetry. Eventually, the ambiguity left by supersymmetry does not really express itself in the X - Y split, but rather in the disconnected left- and right-sided sector and the resulting freedom in $\vec{\mu}_L^{(0)}$ and $\vec{d}_L^{(0)}$.

4.3 Non-Supersymmetric States

It was recently conjectured [3] that some non-supersymmetric but nevertheless extremal black holes might be identified with non-BPS states of the heterotic string. In particular, since the bosonic solution by itself does not particularly distinguish between left- and right-sided gauge fields, black holes may be constructed that are uncharged under the graviphotons, but charged under the left-sided vector fields, and which satisfy the extremal condition $M^2 = Q_L^2/8$ [38]. In this non-supersymmetric case it is no longer possible to make general statements about the dipole moments carried by the bosonic solution. Nevertheless, it turns out that the fermion zero mode contributions have a very general form for any black hole

solution, independent of the specifics of the solution. Thus we may extend the above techniques to the analysis of gyromagnetic ratios of non-supersymmetric black holes. This then allows a further comparison of such states with the heterotic string.

For non-supersymmetric black holes, we are no longer guided by Killing spinor equations. However it is still possible to examine the supersymmetry variations in the asymptotic regime. In this case, since the variations are local expressions, it is clear that the zero-mode-generated quantities, $\delta\delta(\dots)$, can only depend on the mass and asymptotic charges of the solution. Without supersymmetry to couple various fields together, we define the additional charges δ , N and \mathcal{R}^{Ia} according to

$$\begin{aligned}\hat{g}_{ij} &\sim \delta_{ij}(1 + \frac{2\delta}{r}) \\ e^{-\eta} &\sim 1 + \frac{2N}{r} \\ V_L L \partial_i V_R^T &\sim \mathcal{R} \frac{\hat{x}_i}{r^2},\end{aligned}\tag{4.23}$$

where the metric retains the form (3.2). In particular, we have picked an isotropic form of the asymptotic metric, with spatial components $g_{ij} \sim -\hat{g}_{ij}/g_{00} \sim \delta_{ij}(1 + 2(M + \delta)/r)$. Using these definitions, as well as the standard electric and magnetic charges, results in the zero mode spin

$$\vec{S} = -\frac{1}{4}\bar{\epsilon}\vec{\gamma}i\gamma^5[(M_1 - \frac{1}{2\sqrt{2}}\gamma^0 Q_R \cdot \Gamma) + (M_2 + \frac{1}{2\sqrt{2}}i\gamma^5\gamma^0 P_R \cdot \Gamma)]\epsilon, \tag{4.24}$$

as well as the electric and magnetic dipole moments

$$\begin{aligned}\delta\delta\vec{\mu}_R^a &= \frac{1}{\sqrt{2}}\bar{\epsilon}\gamma_0\vec{\gamma}i\gamma^5\Gamma^a[M_2 + \frac{1}{2\sqrt{2}}i\gamma^5\gamma^0 P_R \cdot \Gamma]\epsilon \\ \delta\delta\vec{d}_R^a &= \frac{1}{\sqrt{2}}\bar{\epsilon}\gamma_0\vec{\gamma}\Gamma^a[M_1 - \frac{1}{2\sqrt{2}}\gamma^0 Q_R \cdot \Gamma]\epsilon \\ \delta\delta\vec{\mu}_L &= -\frac{1}{4}\bar{\epsilon}\vec{\gamma}i\gamma^5[Q_L - \sqrt{2}\gamma^0\mathcal{R} \cdot \Gamma]\epsilon \\ \delta\delta\vec{d}_L &= \frac{1}{4}\bar{\epsilon}\vec{\gamma}i\gamma^5[P_L - \sqrt{2}i\gamma^5\gamma^0\mathcal{R} \cdot \Gamma]\epsilon,\end{aligned}\tag{4.25}$$

where $M_1 = \frac{1}{2}(M + N)$ and $M_2 = \frac{1}{2}(M - N + \delta)$. While these expressions are similar to those for the intermediate multiplet ansatz, they are nevertheless quite general. We see that each dipole moment receives a contribution from two terms which are precisely related only in the supersymmetric case. One thing to note is that no dependence on the classical (Kerr) angular momentum of the black hole appears in the above expressions. This is consistent with the supersymmetric black hole results, where one can simply combine the zero-mode-generated dipole moments with their classical counterparts.

In order to study the electrically charged non-supersymmetric $M^2 = Q_L^2/8$ black hole, we may set $\beta = 0$ and take the limit $\alpha \rightarrow \infty$ in Eqn. (3.8). The resulting black hole has charges

$$Q_L = -2\sqrt{2}M\hat{\ell} \quad Q_R = -2\sqrt{2}M\hat{n}\tanh\gamma \tag{4.26}$$

and only a left-sided magnetic dipole moment, $\mu_L = -2\sqrt{2}L\hat{\ell} = (Q_L/M)L$. Here $\hat{\ell}$ is a 22-component unit vector labeling the left-sided $U(1)$ that is active. From the zero modes, we find in particular $\delta\delta\vec{\mu}_L = -2\sqrt{2}\vec{S}\hat{\ell} = (Q_L/M)\vec{S}$ as well as $\delta\delta\vec{\mu}_R = 0$, where the spin \vec{S} created by the zero modes is

$$\vec{S} = -\frac{1}{4}M\bar{\epsilon}\vec{\gamma}i\gamma^5[1 + (\tanh\gamma)\gamma^0\hat{n} \cdot \Gamma]\epsilon. \quad (4.27)$$

This results in the especially simple picture that $g = (2, 0)$ for *all* states in the long supermultiplet corresponding to this black hole. For the electric dipole moments, we find

$$\begin{aligned} \delta\delta\vec{d}_R^a &= \frac{1}{\sqrt{2}}M\bar{\epsilon}\gamma_0\vec{\gamma}\Gamma^a[1 + (\tanh\gamma)\gamma^0\hat{n} \cdot \Gamma]\epsilon \\ \delta\delta\vec{d}_L &= \frac{1}{\sqrt{2}}M\hat{\ell}\bar{\epsilon}\vec{\gamma}[1 + (\tanh\gamma)\gamma^0\hat{n} \cdot \Gamma]\epsilon. \end{aligned} \quad (4.28)$$

In particular, taking the limit $\gamma \rightarrow \infty$ reproduces the supersymmetric results for the Kaluza-Klein black hole, where in this case $\delta\delta\vec{d}_L = 0$ holds on zero-mode spinors satisfying Eqn. (3.17).

Based on the $M^2 = Q_L^2/8$ extremal condition, for this black hole to correspond to a heterotic string state, the latter must have $N_L = 1$ and $N_R = (Q_L^2 - Q_R^2 + 1)/2$. Thus from the elementary string point of view, the Kerr angular momentum and quantum spin both originate from the right side of the string, resulting in the gyromagnetic ratios $g = (2, 0)$, in perfect agreement with the black hole calculation. We also note that the black hole entropy using the stretched horizon approach [22] agrees with the degeneracy of elementary string states. On the other hand, the appearance of non-zero $\delta\delta\vec{d}_L$ would be somewhat of a surprise because of its likely origin from a left-sided Ramond sector, which is of course absent in the heterotic string. However, this term actually vanishes for $Q_R = 0$, corresponding to $\gamma = 0$, since in fact $(\bar{\epsilon}\vec{\gamma}\epsilon) = 0$ for spinors satisfying a ten-dimensional Majorana condition. What this suggests is that only the non-supersymmetric extremal black holes that are completely uncharged with respect to the graviphotons [38] may possibly be identified with elementary heterotic states. However, since the stringy origin of electric dipole moments is not totally understood yet, it is too early to reach a definite conclusion.

5 Conclusion

The original motivation for examining electric and magnetic dipole moments of extremal black holes was to provide a further test of the black holes as elementary string states conjecture. We have found out, however, that for short $N = 4$ multiplets the dipole moments were completely fixed by supersymmetry. Thus in this case the gyromagnetic ratios do not provide a true test of the conjecture, as the result is guaranteed by supersymmetry. Nevertheless, it is reassuring to see that the correct values of (g_L, g_R) arise from two completely different derivations—the string formula [13] on the world-sheet, and the supersymmetry approach in space-time. We have also made clear the connection between superspin of the black hole multiplet and the spin originating from the left side of the heterotic string as well as between the zero-mode spin and the right side of the string.

A somewhat surprising result of the analysis for short multiplets was the appearance of electric dipole moments for graviphoton couplings to electric black holes. Since these electric dipole moments are only present for the spin $L \pm 1/2$ superpartners of the bosonic spin L solution (assuming the standard electric black hole of [14]), from a string point of view their appearance seems to be related to the Ramond sector of a supersymmetric worldsheet. Using heterotic/Type II duality, we find an intricate structure of electric and magnetic dipole moments on both sides of the duality map consistent with this interpretation. The prospect of understanding how electric dipole moments arise from the worldsheet point of view is currently under investigation. Until this is completed it is also difficult to see how black holes with intrinsic electric dipole moment [51] could possibly fit in with string states.

Turning to intermediate multiplets finally allow a test of the strings and black holes conjecture. In this case, however, the comparison must be made to a Type II string, since the elementary heterotic string has no intermediate states. Based on supersymmetry, we find a natural interpretation of an intermediate multiplet black hole as a combination of separate electric and magnetic states in the heterotic picture, with both states independently preserving half of the supersymmetries, yet having only a quarter preserved in common. Because supersymmetry leaves several dipole moments undetermined, we examine the properties of known dyonic black hole solutions. Mapping over to the Type II picture, we find agreement with elementary $\tilde{N}_R = 1/2$ (or $\tilde{N}_L = 1/2$) string states.

Finally, we have given further evidence for the possible identification of some non-supersymmetric extremal black holes with elementary heterotic states having $N_L = 1$. While not protected by supersymmetry, this result may nevertheless indicate the presence of some hidden protection mechanism in M-theory. However, since this black hole does not map into an elementary Type II state, little further insight is gotten from the dual picture.

In this paper we have often combined the separate notions of black holes as elementary string states and string/string duality. In doing so, we have given some further support for both ideas. Additionally, we have seen how well both conjectures complement each other and how useful it can be to study black holes in various dual pictures. Although most dyonic black holes cannot be elementary in any point of view, our results are consistent with black holes as strings whenever its charges allow it to be elementary in an appropriate picture. This outcome suggests an M-theory approach, in which no single black hole interpretation is particularly more fundamental than the other.

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